Tail Risk – an introduction

Presented by: Claus Madsen

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VaR – What is VaR? - I

Definition:

VaR/Risk Capital is defined as the maximum possible loss for a given position or portfolio within a known confidence interval over a specific time horizon.

VaR can play three important roles within financial institutions:

- it allows risky positions to be directly compared and aggregated
- it is a measure of the economic or equity capital required to support a given level of risk activities
- it allows the management to make returns from diverse risky business directly comparable on a risk adjusted basis

VaR – What is VaR? - II

- VaR is more formal defined as the negative quantile of the P/L distribution, that is VaR is defined contingent on 2 arbitrarily chosen parameters:
  - A confidence level α, which indicates the likelihood than we will get an outcome that will be no worse than our VaR
  - A holding period (horizon)

Method for Calculating VaR

- Before going further let us formally define VaR. VaR can be expressed as:

$$VaR = -q_p$$

- In VaR we focus on the worst p% outcomes - the tail of the return distribution (P/L)
  - If we have a confidence level α and set p = 1 - α and if q_p is the p-quantile of the P/L distribution over some holding period, then VaR of the portfolio's P/L at that confidence level and some holding period is given by the above equation!

VaR and Regulatory Capital

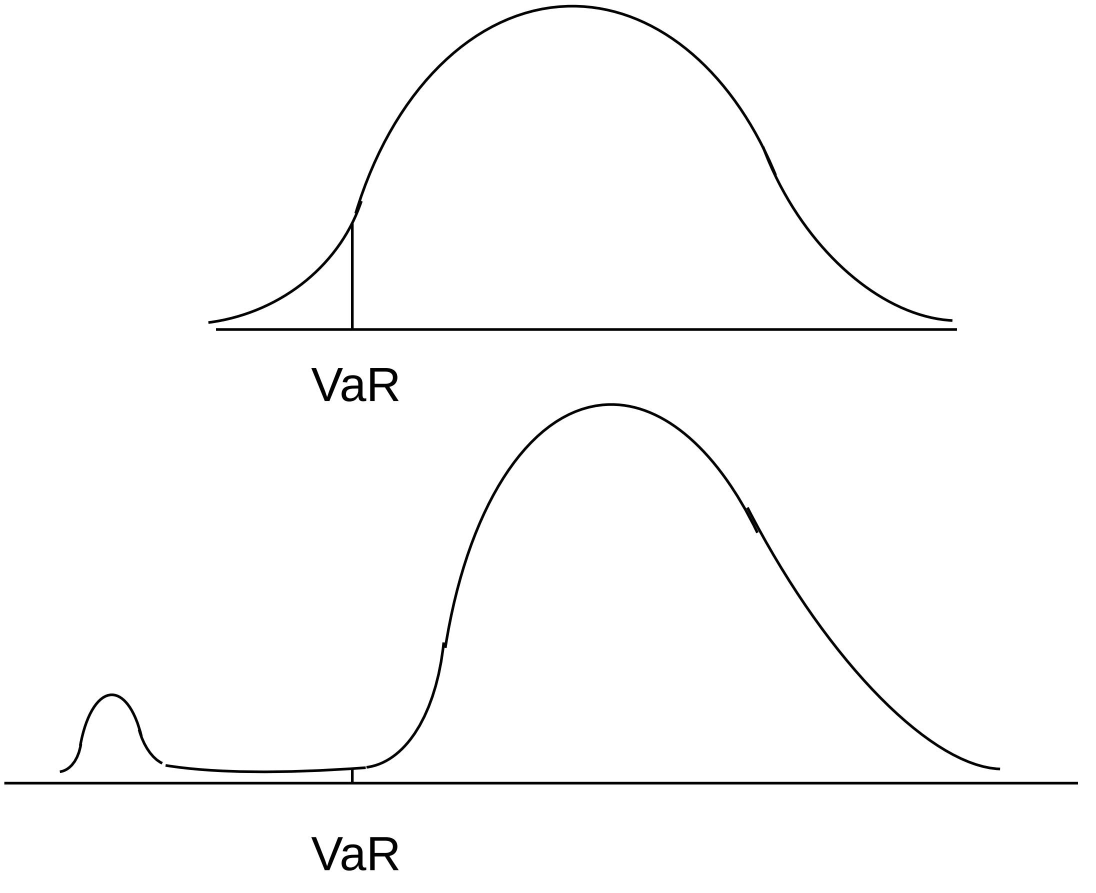
- Regulators base the capital they require banks to keep on VaR
- The market-risk capital is k times the 10-day 99% VaR where k is at least 3.0
- Under Basel II, capital for credit risk and operational risk is based on a one-year 99.9% VaR

- Remark: Changes to ETL is under way…..

Choice of VaR Parameters

- Time horizon should depend on how quickly a portfolio can be unwinded. Bank regulators in effect use 1-day for market risk and 1-year for credit/operational risk
  – Fund managers often use one month
- Confidence level depends on objectives. Regulators use 99% for market risk and 99.9% for credit/operational risk.
- A bank wanting to maintain a AA credit rating will often use confidence levels as high as 99.97% for internal calculations.

Distributions with the same VaR but….



Definition (Wikipedia)

- Tail risk is the risk of an asset or portfolio of assets moving more than 3 standard deviations from its current price in a probability density function
  - This is often estimated using normal statistical methods for calculating the probability of changes in the price of financial assets

CVaR, Tail-Loss and ETL - I

- CVaR is defined as the average of the worst $100(1-\alpha)\%$ of loses, that is:

$$CVaR_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_p dp$$

- If the P/L distribution is discrete, then CVaR can be expressed as:

$$CVaR_\alpha = \frac{1}{1-\alpha}\sum_{p=0}^{\alpha}[\text{pth highest loss}] \text{ x } [\text{probability of pth highest loss}]$$

- That is: CVaR is the probability-weighted average of the tail losses – which suggest that CVaR can be estimated as an average of ”tail VaRs”

CVaR, Tail-Loss and ETL - II

Under a Normal Distribution assumption we can express CVaR as:

(1) $$CVaR = P\left(\sigma\sqrt{h}\,\frac{\varphi(z_\alpha)}{1-\alpha} - \mu h\right)$$

Where is φ the standard normal density function.

Remark: CVaR is what is termed a coherent risk-measure whereas VaR is not – see next slide!

Coherent Risk Measure - I

- What does we mean by a Coherent Risk Measure?
- By a Coherent Risk Measure we mean a Risk Measure that satisfies the following 4 requirements:

1: Monotonicity: $Y \geq X \Rightarrow rm(Y) \leq rm(X)$

2: Subadditivity: $rm(X+Y) \leq rm(X) + rm(Y)$

3: Positive homogeneity: $rm(hX) = h\,rm(X)$ for $h > 0$

4: Translational invariance: $rm(X+n) = rm(X) - n$

for some certain amount n

- Remark: Properties 1, 3 and 4 are well-behavedness properties to rule out strange outcomes. The most important property is the sub-additivity property!

Coherent Risk Measure - II

- The sub-additivity property tell us that a portfolio made up of subportfolios will have a risk that is not more than the sum of the risks of the subportfolios
  - This reflects an expection that when we aggregrate individual risk, they diversify or, at worst, do not increase – the risk of the sum is always less than or equal to the sum of the risks

- This sub-addivity property spells trouble for VaR as VaR is not sub-additive – see next slide for an example!

Coherent Risk Measure - III

- We have 2 identical bonds, A and B. Each default with probability 4%, and we get a loss of 100 if default occurs, and a loss of 0 if no default occurs. The 95% VaR for each bond is therefore 0, so VaR(A) = VaR(B) = VaR(A) + VaR(B) = 0. Let us now suppose that defaults are independent. Elementary calculations now establish that we get a loss of 0 with probability $0.96^2 = 0.9216$, a loss of 200 with probability $0.04^2 = 0.0016$, and a loss of 100 with a probability of $1 – 0.9216 – 0.0016 = 0.0768$. That is VaR(A+B) = 100. Thus VaR(A+B) = 100 > 0 = VaR(A) + VaR(B), and the VaR violates the sub-additivity property – hence VaR is not a Coherent Risk Measure!

Coherent Risk Measure - IV

- It is actually only possible to make VaR satisfy the sub-additivity property by imposing restrictions on the form of the P/L distribution
- More precisely it turns out that one has to impose the severe restriction that the P/L distribution is elliptical
- However, in the real world non-elliptical distributions are the norm rather than the exception
- **Remark:** The Normal Distribution belongs to the class of elliptical distribution, so imposing this restriction will make VaR a Coherent Risk Measure

Coherent Risk Measures revisited – Spectral Risk Measures

- A spectral risk measure assigns weights to quantiles of the loss distribution
- VaR assigns all weight to Xth quantile of the loss distribution
- CVaR assigns equal weight to all quantiles greater than the Xth quantile
  – For that reason CVaR is sometimes referred to as a risk-neutral Risk Measure!
- For a coherent risk measure weights must be a non-decreasing function of the quantiles

Coherent Risk Measures revisited – Distortion Risk Measures

- One candidate is the renowned Wang(2000) transform, which can be expressed as:

$$DRM(PL,\alpha) = \Phi[\Phi^{-1}(PL) - \Phi^{-1}(\alpha)]$$

- For the market price risk being equal to: $\Phi^{-1}(\alpha)$

- This distortion function is everywhere continuous and differentiable. The continuity ensures that is produces coherent risk measures.

Mitigating Tail Risk

- Mitigating Tail Risk Exposure:
  - Investors concerned about tail risk can apply some practical rules of thumb when considering the sizing of allocations and the prudent use of leverage
    - If you cannot take the “pain,” hold less equity
  - Option strategies, including addressing their long-term Cost
    - Out-Of-The-Money Puts (Example: At the end of 2010 one-year put options on the S&P 500 that were 15% out of the money cost approximately 4%, a premium implying a breakeven market decline of over 19% - damn expensive!)
  - Strategies that introduce asymmetric return patterns
    - “Tail Risk” funds or Black Swan fund....(Think of it as buying insurance at the black jack table. The dealer has an ace showing, you think he might have black jack. You buy the insurance and the dealer flips up a King. You lose your bet but you get a little bit back on the side. You live to gamble another day – Survival!)
  - More later……