RenteDyk™ - The new boy in class

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Agenda

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Definition

- The Components:
  - Coupon is a function of the 10Y CMS (Constant Maturity Swap) rate + a fixed spread
  - A sold Ratchet cap on the 10Y CMS rate
  - A sold Bermuda call option with a strike equal to 105

- The future coupon can only fall – never rise! Structures with similar characteristics as RenteDyk™ are usually referred to as One-Way-Floaters

- Bermudan options are a well known product – but what is a Ratchet? – see next slide....

What is a Ratchet cap?

- A Ratchet option (sometimes called a Reset option or Cliquet option) is a series of consecutive forward starting options. The first is active immediately, the second becomes active when the first one expires etc. Each option is struck at-the-money when it becomes active – that is the future strikes are stochastic!

- Ratchet features can be incorporated into other structures – like for example RenteDykTM. (They are frequently encountered in structured equity products)

- The payoff from a Ratchet Cap can be expressed as:

$$Payoff_T = (T - t)Max[R(T) - C(t), 0]$$

- This implies that C(T) can be expressed as (assuming a zero spread):

$$C(T) = (T - t)Min[R(T), C(t)]$$

Valuation of Ratchet Options - I

- For the valuation of Cliquet Options (the terminology normally used in the equity market) – we have the following methods available:

- (European Style Cliquet Option) Closed form solution – due to Rubenstein (1991) - price as a series of forward-starting options. More sophisticated techniques where the future strike only change if at the reset date the stock price is below/above (put/call) can be handled by the method of Gray and Whaley (1999)

- A more general approach is the binomial pricing method from Huag and Haug (2001), which relies on the CRR binomial model

- Monte Carlo simulation is however the most natural approach for the pricing of Cliquet Options, due to the path-dependency embedded in the determination of the future strike levels

Valuation of Ratchet Options - II

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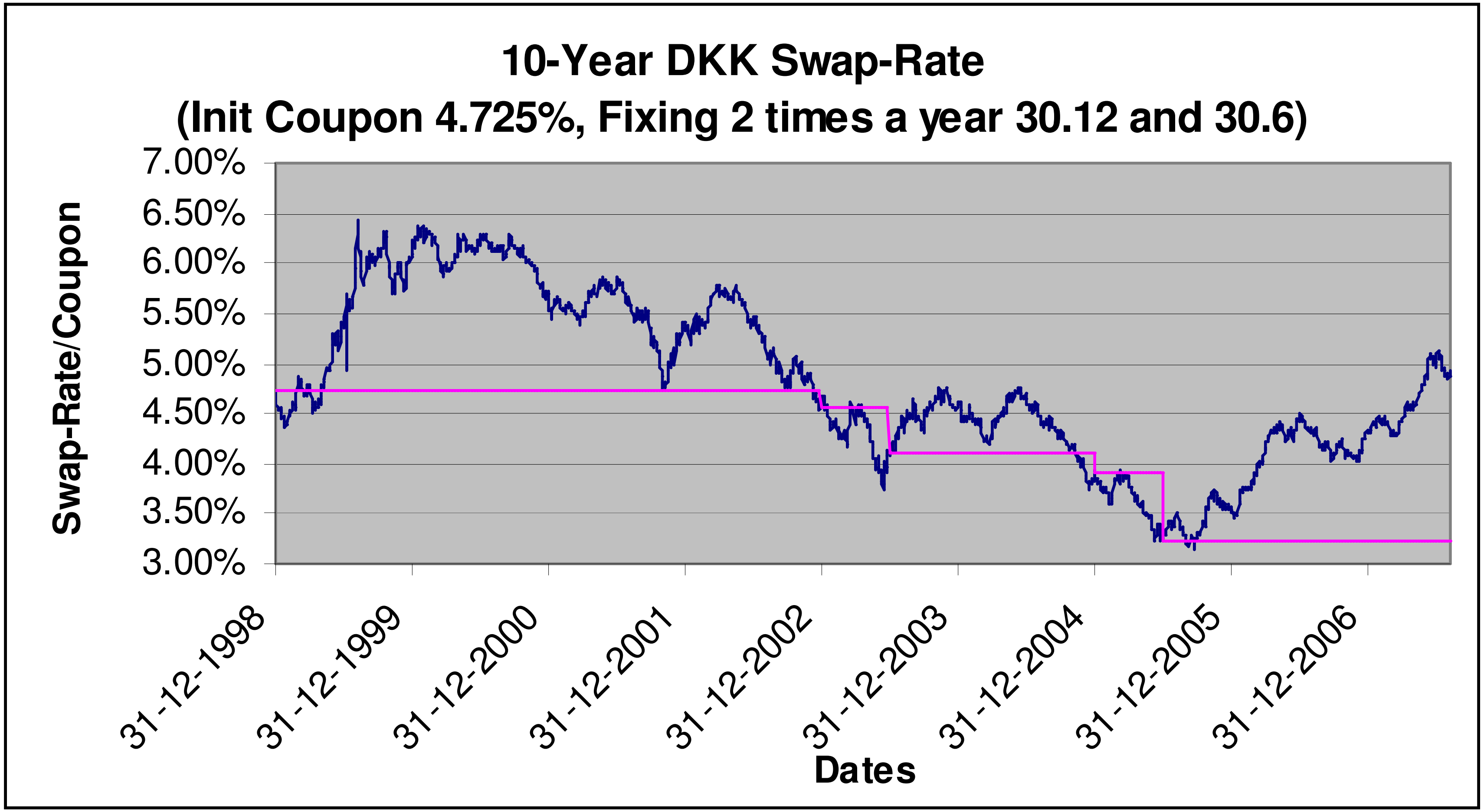
- For Rachet Options (now we are back in the interest-rate world!) the complexity is somewhat greater when it comes to valuation - analytical formulas are hard to come by even in simple cases!). Therefore, for Ratchet Options we would in general need some kind of numerical method and the natural pricing methodology to choose is Monte Carlo Simulation

- However a finite difference method can also be used by adding an extra state-variable

Visualizing the stochastic property of the future strike levels



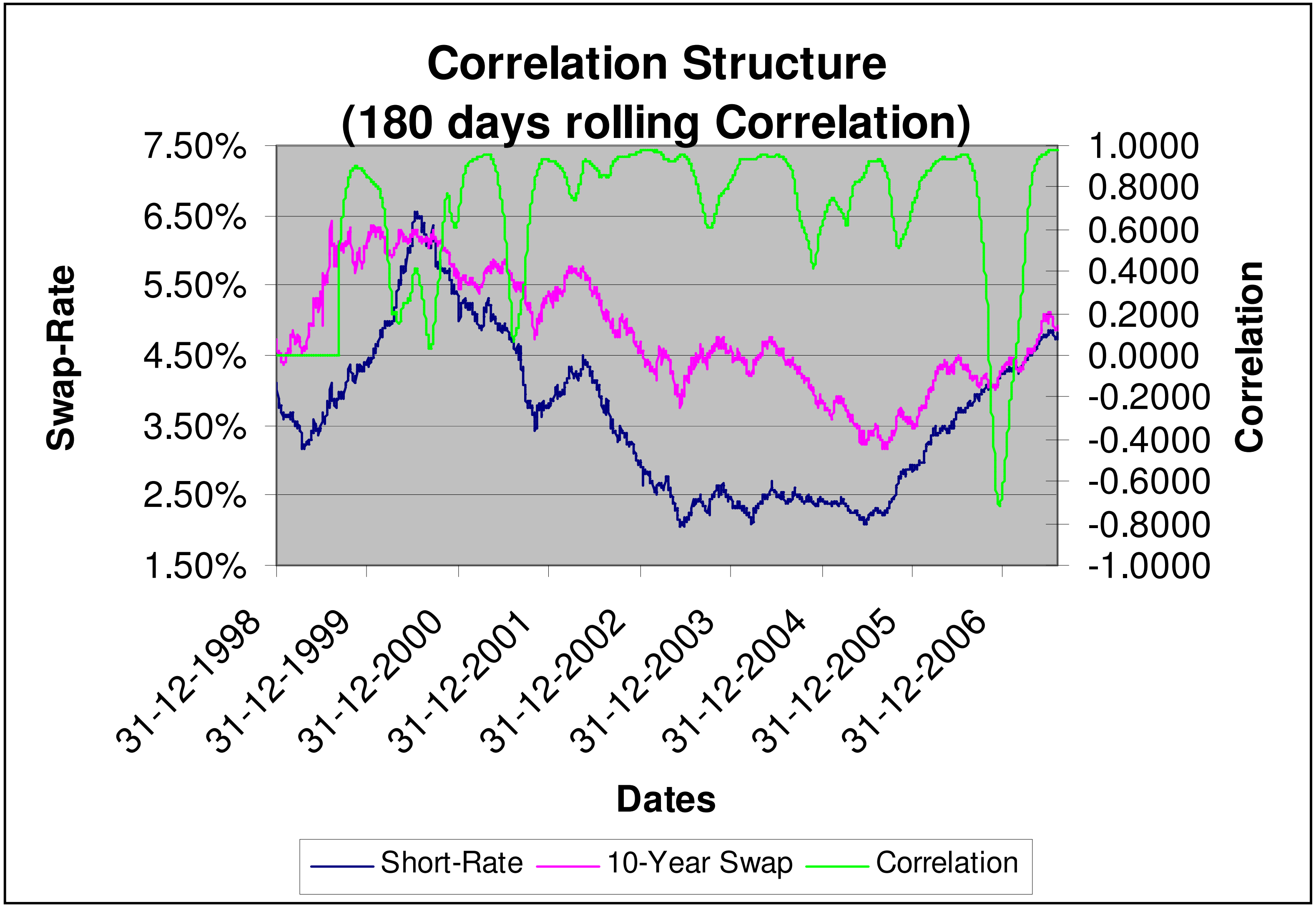
Valuation of Ratchet Options - III

- For this particular type of Ratchet Cap embedded in RenteDyk™ – we have the following additional issues:

  - We need a 2-factor model, due to the fact that the short-rate and the 10Y CMS Rate are not 100% correlated – see next slide....
  - We need to take into account the volatility for At-The-Money Swaptions and over time volatility for Out-Of-The-Money Swaptions due to the fact that we expect to have an increased exposure to lower and lower strike levels as time goes - Volatility Smile dependency!

- These 2 factors clearly add complexity to the pricing of RenteDyk™

Correlation between
The Short-Rate and The 10-CMS Rate (avg Corr: 0.79)



The Pricing of RenteDyk™

- What Interest Rate Model to choose?

- In this case it is natural to use the Libor Market Model as it is straightforward to extend to multiple factors – more precisely the Extended Libor Market Model. Extended, as we need to include the Volatility Smile. Candidates are for example:
  - The CEV version of Andersen and Andreasen (2000) – Alternatively the Displaced Diffusion version (Rebonato (2001))
  - Lognormal-Mixture Model of Brigo and Mercurio (2000)
  - The stochastic-Volatility model of Andersen and Brotherton-Ratcliffe (2001)
  - The SABR model of Hagan, Kumar, Lesniewski and Woodward (2002)

  - A short-rate model is not the obvious choice – let us however still pursue that possibility for the following practical reasons: Speed of calculation, comparison with other mortgage bonds – which normally are valued using a 1-factor short-rate model - simplicity and therefore transparency

The Pricing of RenteDyk™ – Short-Rate Models - I

- The most popular short-rate model in the market is the Hull-White model:
  - Our Hull-White implementation for RenteDyk™ is a Monte-Carlo based pricing method – the standard seems to be a lattice implementation, this (interesting) discussion will however not be pursued here....

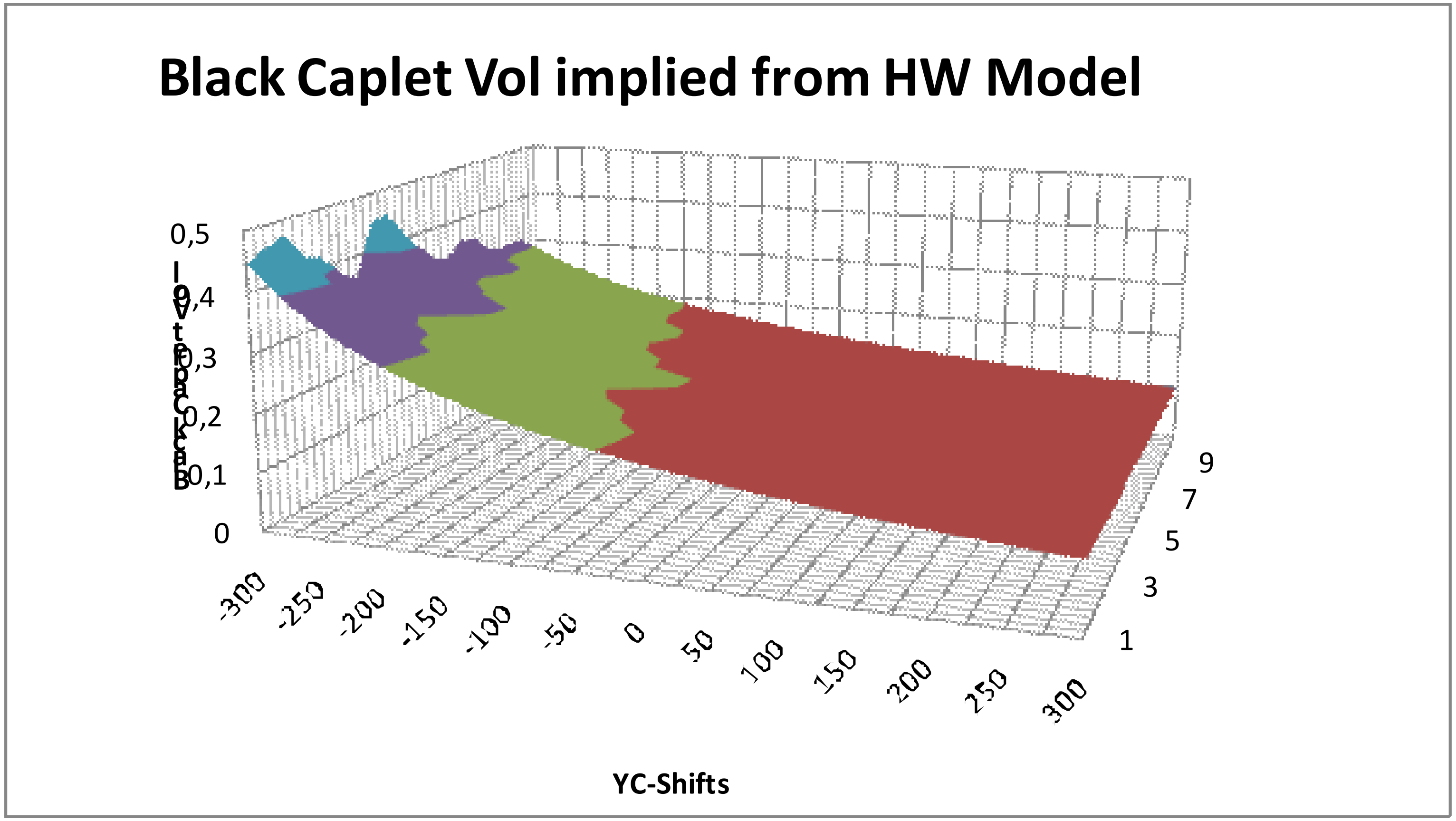
  - A few interesting issues for the Hull-White models implication for the volatility sensitivity and for volatility smiles will be given here....

  - Some stylized facts 1:
    - The Hull-White model has that property that there is a negative correlation between (caplet) volatility and interest rates (a desirable feature) – i.e. the (caplet) volatility increases when rates are falling, the reason being that we have the following formula:

$$v_{T-Caplet} \approx \sigma \frac{1-e^{-\kappa\tau}}{\kappa}\left[1+\frac{1}{\tau F(0,T,T+\tau)}\right]$$

- This effect is shown on the next slide....

Implied Vol Sensitivity for the Hull-White Model



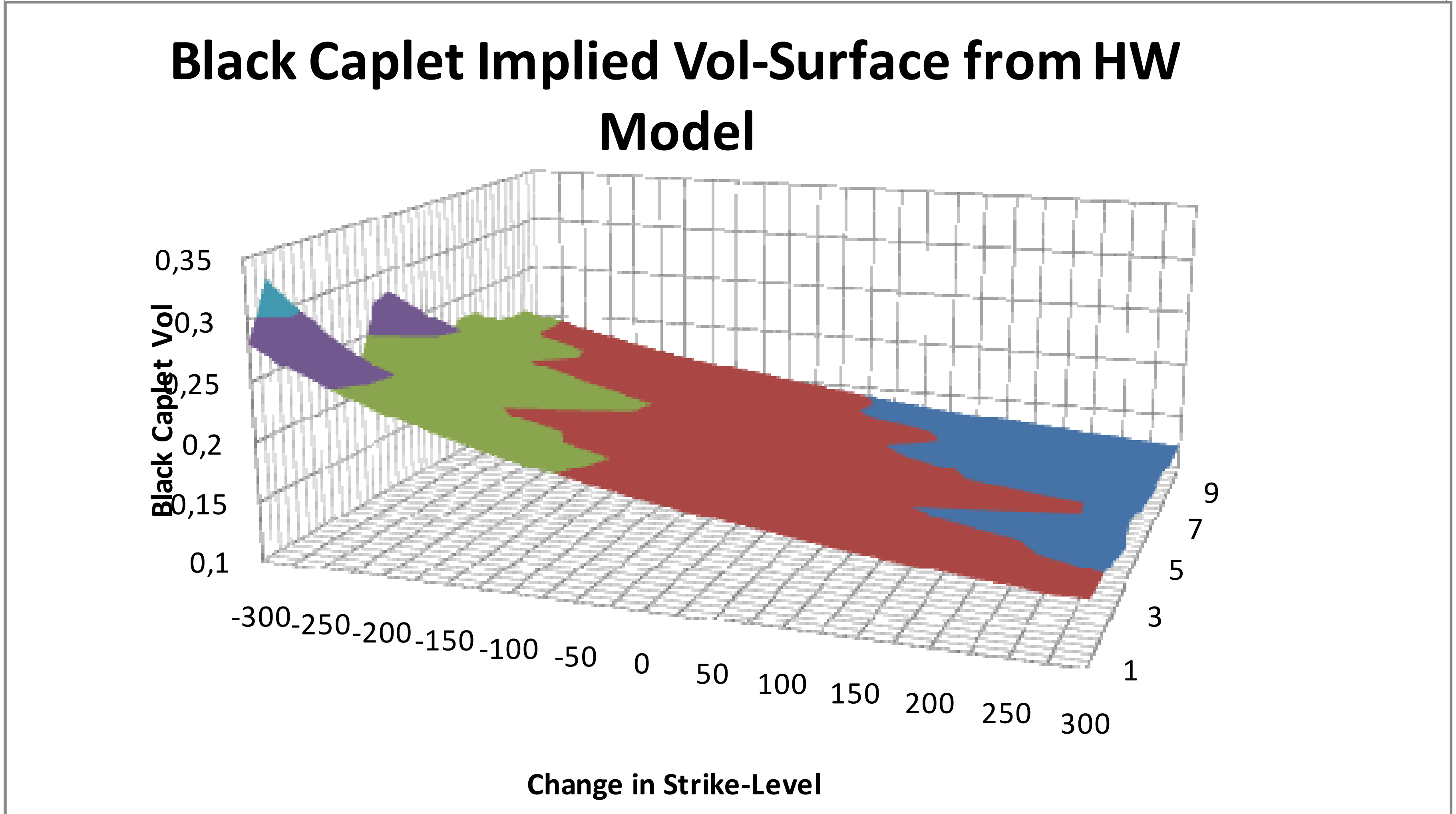
- This figure illustrates this effect which is obviously more pronounced for shorter maturity Caplets than for longer maturity Caplets
- Hint: Black Caplet Vols are given as decimals

The Pricing of RenteDyk™ – Short-Rate Models - II

- Some stylized facts 2:
  - Embedded in the Hull-White model is a volatility smile in the implied Black Caplet volatility
  - This smile is however not changeable with a different parameterization of the Hull-White model as there is no direct control over it – it is embedded in the model
  - In general the slope of this embedded Black volatility smile is small compared to the market
  - There can exist cases where it is not possible to imply a Black Caplet Volatility – given the price obtained from the Hull-White Model – the HW price is unattainable! (this happens for ”high” values of σ in the Hull-White Model)
  - We also have that smirks are not possible in the Hull-White Model

This feature is shown on the next slide....

Implied Volatility Surface for the Hull-White Model



The Pricing of RenteDyk™ – Short-Rate Models - III

- We can now list the following issues connected with using a 1-Factor model – more precisely the Hull-White Model:

  - Even though the model embeds the (desirable) feature that (Black) volatility rises when rates falls – we have limited control over it, the relative slope is nearly constant for all parameterizations. A slightly higher relative slope can though be observed for ”short” maturities and for mean-reversion => 0
  - It is not possible to fit the ATM humped Cap Volatility curve – as only ”small” humps are normally observable. A large hump is only possible in case the YC is decreasing – which is not the norm...
  - The embedded volatility smile in the Hull-White model has a too small slope – the relative slope is approximately constant across different parameterizations
  - There can exist cases where it is not possible to Imply a Black Caplet Volatility – given the price obtained from the Hull-White Model – the HW price is unattainable! (happens for ”high” values of σ in the Hull-White Model)
  - Smirks are not possible in the Hull-White Model
  - As it is a one-factor model the 10-year rate will be a deterministic function of the short-rate

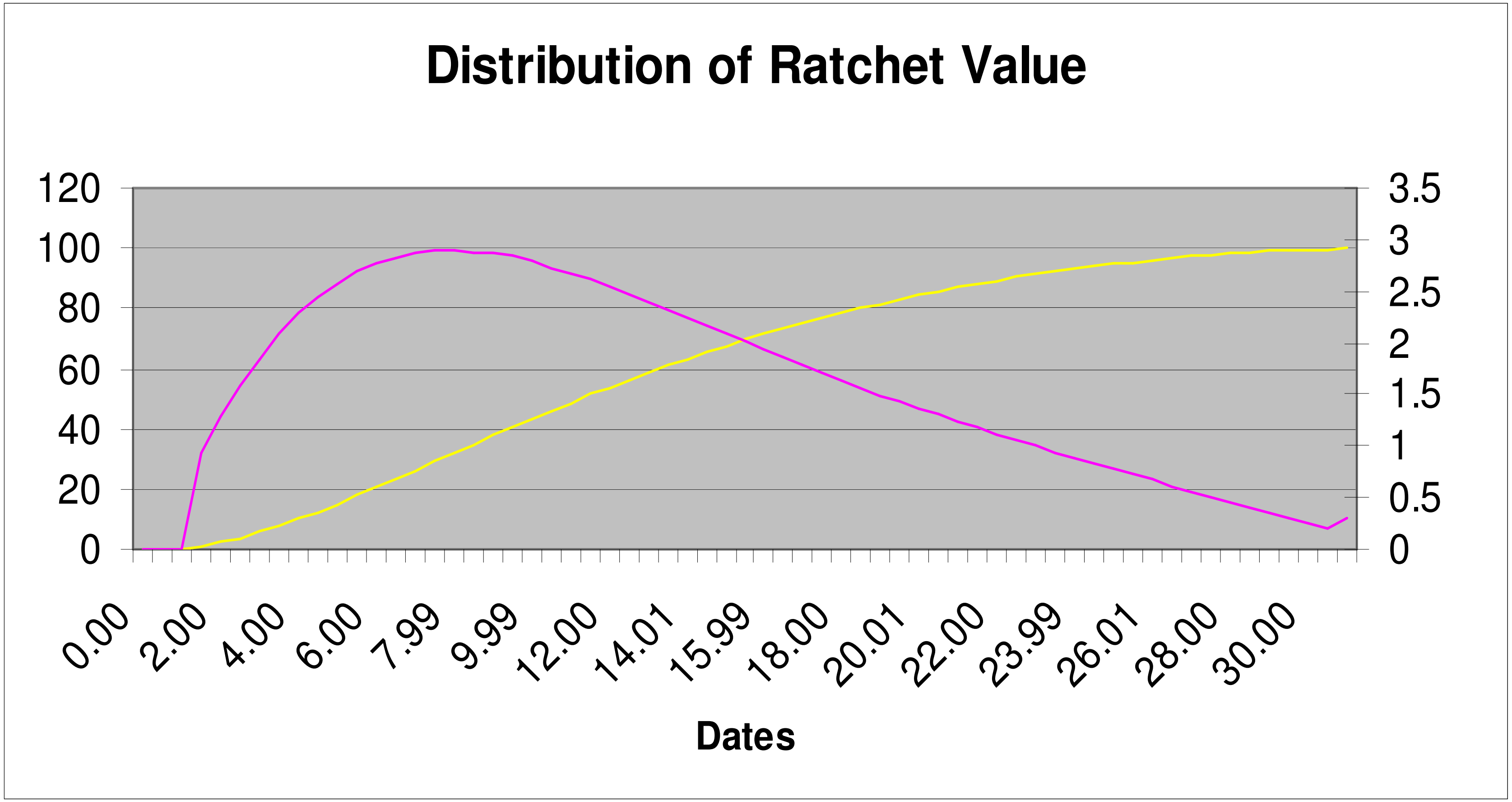
- Hint: When using a simpler model than theoretically is required knowing the limitations is necessary...

Risk-Measures....

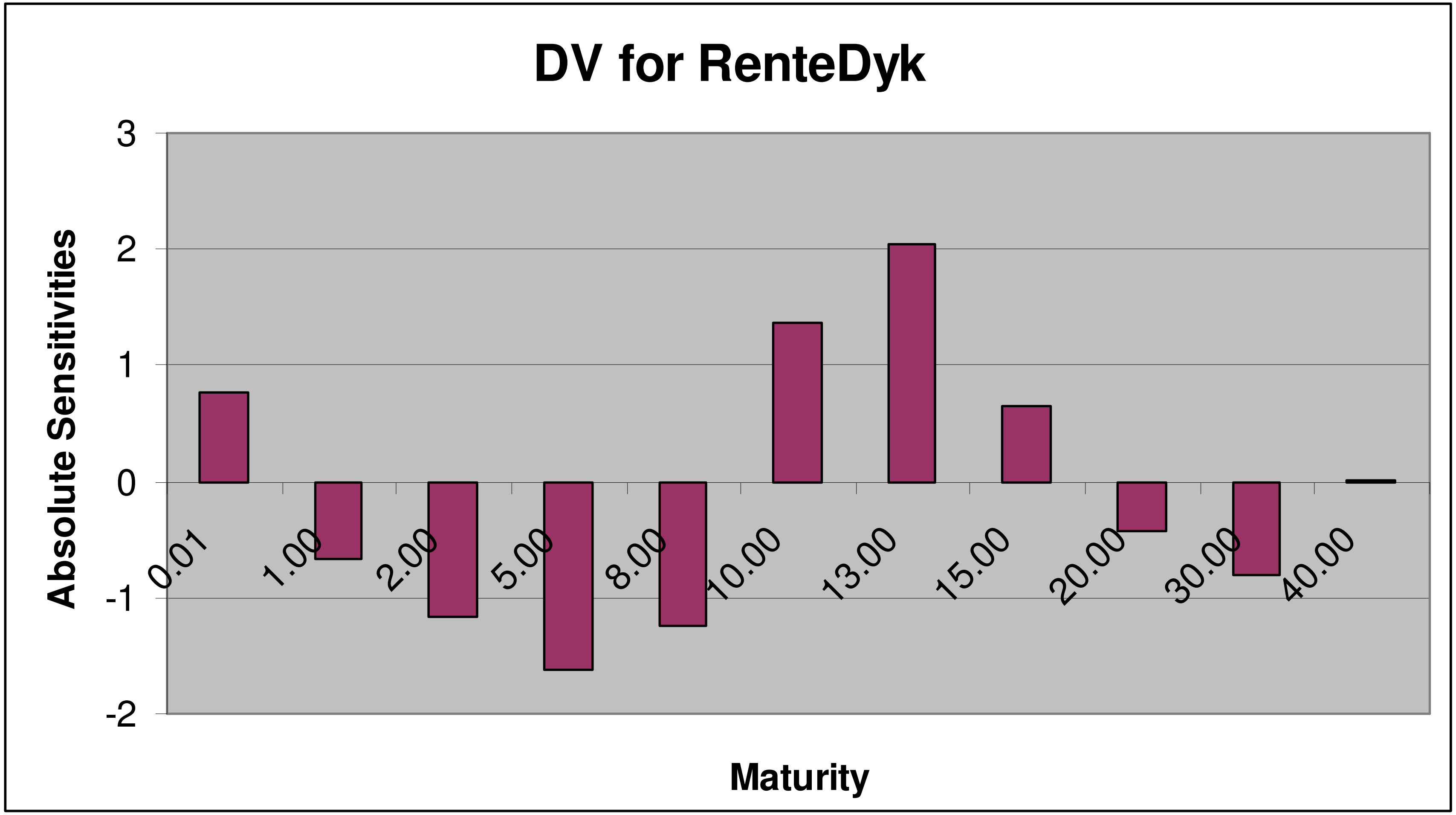
- We will here look at 3 issues:
  - The distribution of Ratchet prices across the maturity spectrum
    - On slide 16 it can be seen that the most of the value of the Ratchet Option embedded in RenteDyk^{TM} is located around the 10-year maturity point...
  - The delta-vectors
    - On slide 17 the distribution of sensitivies across the maturity spectrum – Key-Rate sensitivities – are shown
  - The stability of duration measures for different increments
    - On slide 18 it is obvious that duration is a function of the increment, that is duration is scale dependent!

- Besides that we will – from a theoretically point – discuss the stability of duration measures over time

Ratchet Price Distribution



Delta Vectors – Distribution of Sensitivities



Duration as a function of the Increment. Unit = 100 Bp

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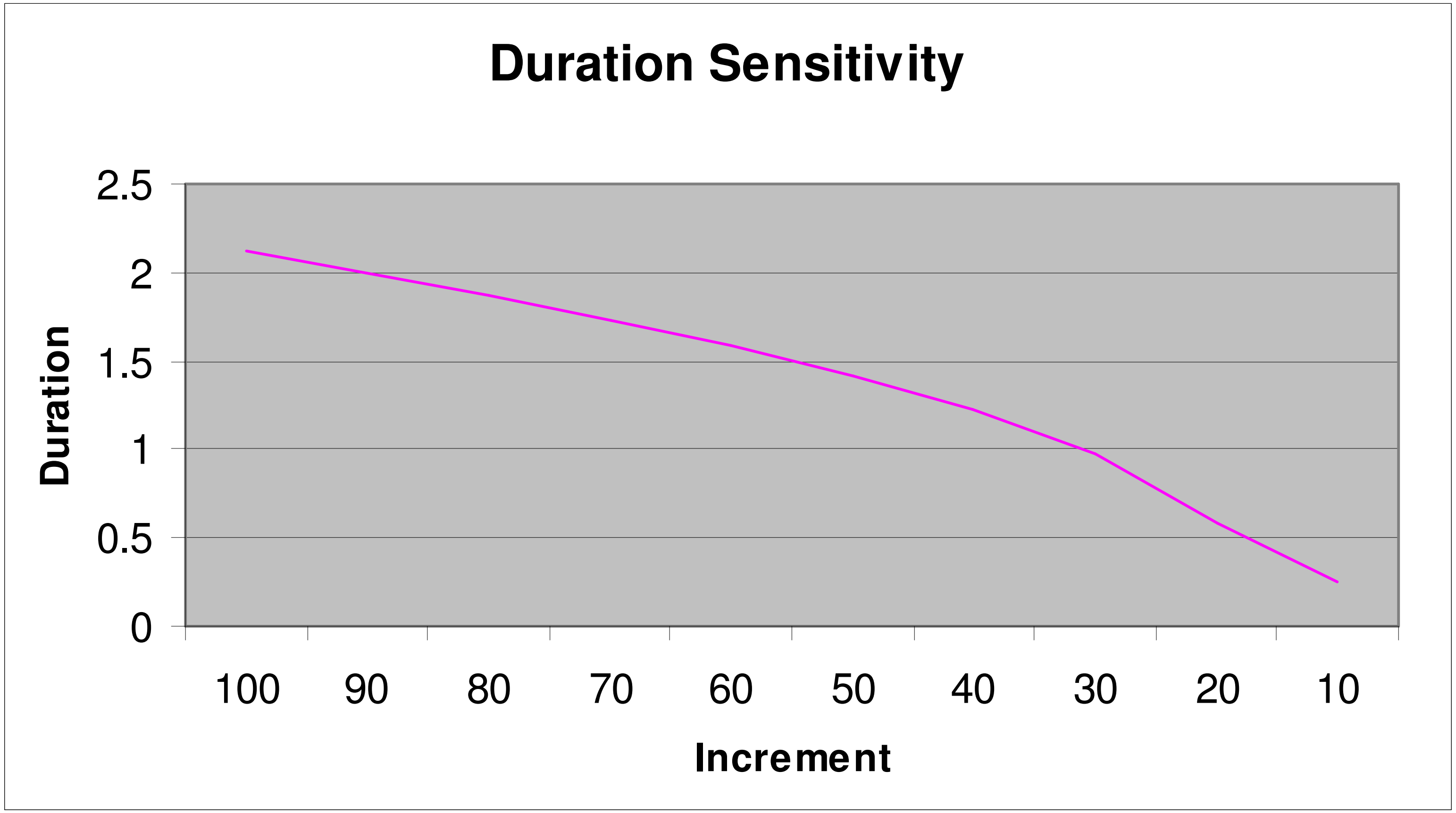

Risk-Measures – The nature of Ratchet Options

- Wilmott (2002) highlights the danger of volatility risk in Rathets, as it can be shown that the gamma of a Cliquet option is the sum of gamma values for regular options because the gamma of a forward-starting option is zero before starting time. This may create the impression that risk-management is easy in this case. However, for this type of option, hedging can be quite complex because delta, vega and gamma have discontinuities around reset times!

- The above entails the following:
  - Even under a stable interest-rate environment, risk-measures will behave erratic as we are getting nearer and nearer to the fixing-date – and risk-measures will be discontinous as we pass the fixing-date!

Some closing remarks on Risk-Measures for RenteDyk™

- 1-order Risk-Measures are scale dependent, i.e. it is a function of the selected increment
- Risk-Measures (duration and convexity) will even under a stable interest rate environment behave erratic as we are getting nearer and nearer to the fixing-date – and risk-measures will be discontinous as we pass the fixing-date
- Risk-Measures are discontinous around the fixing-dates
- Vega is remarkably higher than for Capped-Floaters
- Delta-Vectors – in general it can be said that the sum of the Delta-Vectors is not equal to OAS-Price Risk. That is DVs for RenteDyk™ (only) indicate/highlight the relative importance of a shift at different parts of the curve

- So – let's be careful out there...