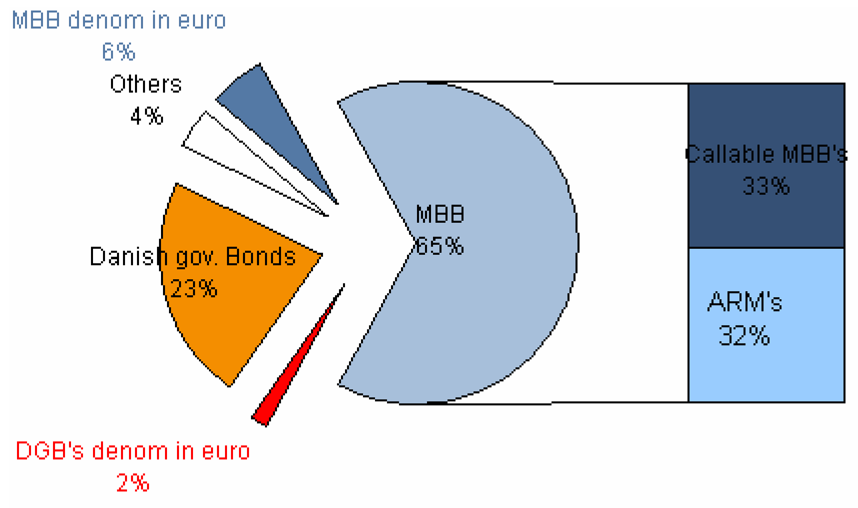
Danish Mortgage Modeling in FinE

Introduction

The Danish bond market is among the largest in the world because of the Danish mortgage bond market, which is significantly larger than the government bond market.



The Mortgage Credit System

The Danish mortgage finance system has existed since 1797, when the first mortgage credit institution was established to provide financing for homeowners who had lost their homes during the Great Fire in Copenhagen in 1795.

The first mortgage credit legislation was made in 1850, but the mortgage system we know as of today is the result of three major reforms during the past three decades: the 1970 reform, the 1980 reform and the present Mortgage Credit Act of 1989[1].

The general structure of the Danish mortgage credit legislation serves to protect mortgage bond investors from losses if debtors should default on their mortgage installments/repayments.

Solvency ratio: The equity must never be less than 8% of the assets. In other words, there is an over-collateral (OC) of at least 8%. In case of bankruptcy, the OC will be used to pay bondholders in full before other creditors of the estate.

Danish mortgage bonds are based on loans against registered mortgages on real estate. The Mortgage Credit Act contains 'a principle of balance' – which is the main pillar for the Danish system for mortgage finance, as it virtually eliminates the funding risk in a mortgage credit institution. The principle of balance states that:

- No bond will be issued without an underlying mortgage
- The debtor side and the creditor side of the institution must balance as a whole (and on series level as well)
- In case of a mortgage bank bankruptcy, then a liquidator will provide that money from debtor payments will be handed over to the investors[2]

Therefore, Danish mortgage bonds have rank of gilt-edged papers.

Mortgage Bonds

Traditionally Danish mortgage bond were issued as fixed-rate callable mortgage bonds based on an annuity principle used to constitute most of the property financing in Denmark. A feature of Danish mortgage bonds (the traditional

[1] See Danske Research (2001).
[2] There has never been a case of default on Danish mortgage bonds.

callable ones) is that the borrower has a right to redeem his/her loan at par (price 100) at any time during the life of the mortgage.

Since the launch of non-callable floating-rate bonds in 1996, the demand for this type of product has been steadily increasing – mainly due to the fact that the yield-curve has been ”steep” (and the rates falling), and furthermore these products made it possible to finance to the short-rate.

In the late 2004 two new financing instruments emerged – namely: A capped floater (callable at strike 105) called RenteMax[3] and a floating to fixed bond product called FlexGaranti[4] (callable at strike 100, when fixed). The main contribution for these two products was to try to combine the benefits of the traditional fixed-rate callable mortgage bond with the benefits for the non-callable floating-rate bonds.

Factors influencing the prepayment behavior

Compared to American callable mortgage bonds there are the following two main differences:

- In America the call provision only allow prepayment of mortgages at par value, it is not as in Denmark possible to buy the bond in the market at the market value and deliver these to the mortgage institution to cancel out
- In America new homeowners cannot take over an existing mortgage (this is possible in Denmark)[5]. This means that prepayments in America will be related to the house-turnover

In Denmark the prepayment incentive is mostly interest-rate dependent. There exist however three cases which can have a big impact on prepayment – and which are not solely interest-rate related:

- Change of tax-rules
- Media campaign
- Issuing of new (re)financing products

The FinE Prepayment Model (FiPaM) is for that reason mainly interest-rate driven.

Available Information

The mortgage credit institutions provide information to investors via Copenhagen Stock Exchange (www.cse.dk).

The table below is a list of the information available:

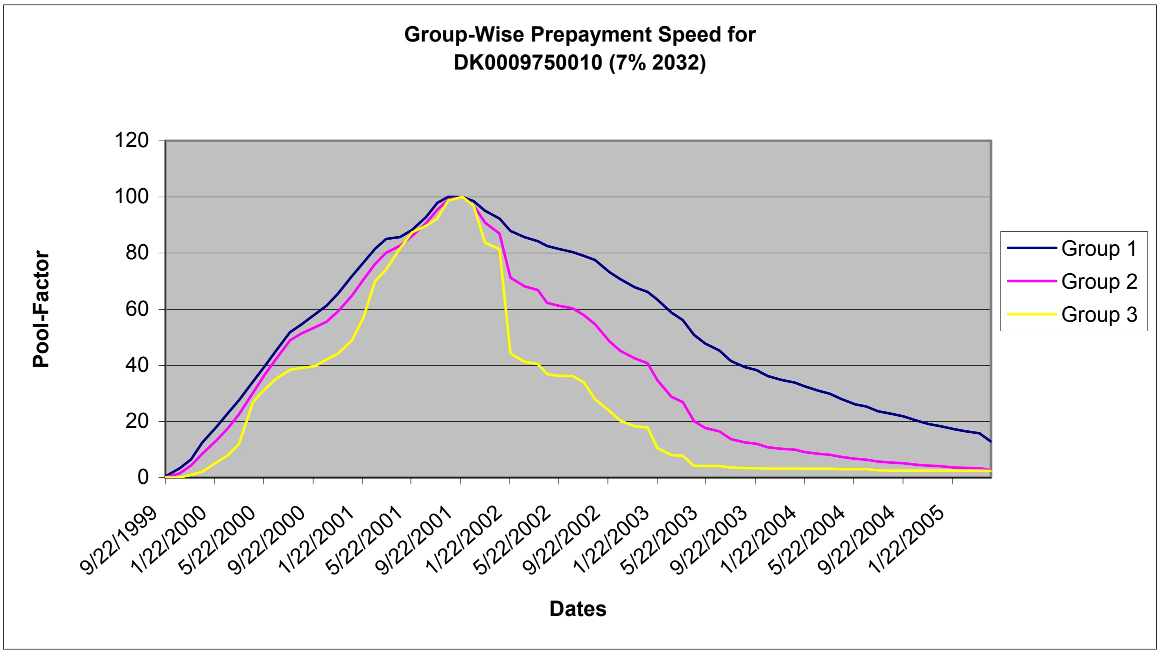
Data	Calculated	Available
Cash flows	Quarterly	13 working days after the payment date
Debtor distribution (five loan interval groups[6] for ”Private” loans and ”Other” loans)	Monthly	Fourth Thursday of every month
Announced prepayments	Every Friday	Tuesday after the calculation day
Published prepayments	Quarterly	Publication date

The FinE Prepayment Model (FiPaM)

Due to the fact that the group ”Private” (private homeowners) over time has become a more professional player in this game (the game of house refinancing[7]) there is no reason to assume that there is any significant difference between the behavior for the average ”Private” borrower and the average ”Other” borrower.

[3] RenteMax is a floating-rate bond with a cap.
[4] FlexGaranti is a floating-rate bond with a knock-in-rate. If knocked-in, the bond becomes a traditional callable mortgage bond with the predefined coupon.
[5] The difference is due to the fact that in Denmark the loan follows the house, whereas in America it follows the person.
[6] These are: 0-0.2 mill, 0.2-0.5 mill, 0.5-1 mill, 1-3 mill and above 3 mill.
[7] One might even call it a national sport.

However, a difference in prepayment speed is inherent in the historical prepayments when comparing loan-sizes. This is exemplified in the graph below:



The difference in prepayment speed can easily be observed by investigating the slope[8] of these curves. The following can be deduced from the graph:

- The prepayment speed for Group 3 is relative more aggressive than the prepayment speed for Group 2 and Group 1 for "high" pool-factors
- For "medium" size pool-factors the conclusion is mixed. In general however, the prepayment speed for "medium" size pool-factors across the Groups are much more agreeable
- For "lower" pool-factors the prepayment speed for Group 1 is relative more aggressive than the prepayment speed for Group 2 and Group 3

These 3 points are what can be termed general observation for Danish mortgage bonds. These are some stylized facts that are expected to be inherent in the specified prepayment model.

For these reasons FiPaM works with 3 debtor groups as follows:

- Group 1: 0-0.5 mill
- Group 2: 0.5-3 mill
- Group 3: above 3 mill

That is - there is no direct distinguishing between "Private" and "Other" loans[9].

From this it can be deduced that the total prepayment for a given bond is given by:

(1) $$\Lambda = \sum_{i=1}^{3} w_i \lambda_i$$

[8] From the point in the graph where all the pool-factors is equal to 100. The reason for the increase in pool-factors in the first part of the graph is due to the 3-year opening period for the bond where issuing is taken place.

[9] However, when determining each of these groups the tax-rate that are used as appropriate for any given group is calculated taken into account the distribution between "Private" loans and "Other" loans. The same applies to the cash-rate and the average loan-size.

Where w_i and λ_i is the weight and prepayment for group i, for $i \in [1,2,3]$. Furthermore we have that the weights sum to one (1). The weights are being calculated using the outstanding bond debt for each of the 3 groups.

Calculating the Gain

The FiPaM belongs to the class of Required Gain Models - which is the market standard.

The Gain is measured as the percentage change in PV after tax, that is:

(2)
$$Gain = \frac{PV_{old} - PV_{new}}{PV_{old}}$$

Where PV_{old} and PV_{new} is respectively the present value (after tax) for the "old" loan and the present value (after tax) for the "new" loan[10].

The following main assumptions are used when performing the above calculation:

- The maturity of the "new" loan is identical to the maturity of the "old" loan
- The loan-type of the "new" loan is identical to the loan-type of the "old" loan
- The number of payment dates a year on the "new" loan is identical to the number of payment dates a year on the "old" loan
- It is assumed that the coupon on the "new" loan is equal to the refinancing-rate
- It is assumed that the "new" loan is a cash-loan, and where the cash-rate is identical to the refinancing-rate
- If the "old" loan has a delayed repayment schedule, then the "new" loan will also have a delayed repayment schedule
- The nominal amount for the "new" loan is calculated using the immediate prepayment at par method[11] - taking into account all the costs that are associated with a refinancing

The above assumptions are what can be termed neutral assumptions (neutral behavior[12]) – as Gain is now fully determined by the change in refinancing-rate.

Determining the refinancing-rate

The determination of the refinancing-rate is linked to the assumed issuing pattern for new mortgage bonds. As a rule of thumb it is assumed that the debtor refinancing structure is a stepwise linear function in the following 4 segments:

- Segment 1: $0 < T \leq 10$
- Segment 2: $10 < T \leq 15$
- Segment 3: $15 < T \leq 20$
- Segment 4: $20 < T \leq 30$

- where T is the remaining time to maturity of the existing loan. This means that if for example the remaining time to maturity of the existing loan is 17 years the refinancing rate will be determined using the yield-to-maturity of the 20-year bond.

As the new loan (in the model) is assumed to have a price equal to 100 and the fact that the new loan by construction is a cash-rate loan, then it follows that the debtor yield-curve will be flat in each of the segments.

From a modeling point of view FiPaM do not work directly with for example a 20-year bond. Instead, proxy instruments are being created along the following lines:

[10] The present value after tax on both the "old" loan and the "new" loan are being calculated using the after tax yield on the "new" loan.
[11] See Madsen (1998).
[12] For example it is not imposed that a refinancing gives rise to a loan with a longer maturity.

- For each of the segments a proxy zero-coupon bond is created with a maturity equal to the duration of the bond given the prevailing debtor yield-curve
- These proxy instruments are being forced market conformed by ensuring that the refinancing-rate derived using the proxy instruments coincide with the ”true” refinancing-rate

Deriving the prepayment-rate λ

The prepayment-rate is for group i being calculated as follows:

(3)
$$\lambda_i = \frac{N(G_i;\mu_i,\sigma_i) - N(Level;\mu_i,\sigma_i)}{1 - N(Level;\mu_i,\sigma_i)}$$

That is the model is using the truncated normal distribution in order to ensure that no prepayments are being observed for ”Returns” below the value given by Level[13].

μ_i and σ_i is respectively the mean ”Return” for group i and the variation around the mean ”Return” for group i.

G_i is a function as follows:

(4)
$$G_i = f\left(Gain_i, PoolFactor_i, YCSlope, YCChange, \frac{Mat_{remaining}}{Mat_{\max}}, \alpha\right)$$

That is the ”Return” is a function of the Gain, the pool-factor, the slope of the yield-curve, the changes in the yield-curve, the ratio of the remaining time to maturity and the maximum time to maturity and the unknown parameter vector α.

Estimation Results

All these unknown parameters are being estimated[14] simultaneously for the total selected data set and for subsets of the total data set. The estimation period is 2 years – namely the years 2003-2005. The bonds used in the estimation are bonds with a coupon between 4 and 8 percent and they cover the whole maturity range – the number of observations is 4368.

For the whole dataset we have run an origo regression[15] on the relationship between the actual prepayments and the estimated prepayments, below are given the results:

Slope	Std. Error	t-Statistics	P-Value	R^2
0.8662	0.00882408	98.1635	0	86.88

This is a very satisfactory result.

However, there is a tendency that the model underestimates high actual prepayments – as can be seen from the value of the Slope-parameter. This tendency is centered round especially one payment date – namely January 2005.

- With respect to January 2005, then there was a high degree of media effect involved here due to the fact that new type of refinancing instruments was introduced (see the section “**Mortgage Bonds**”) – which cannot (and should not) be captured by the prepayment model

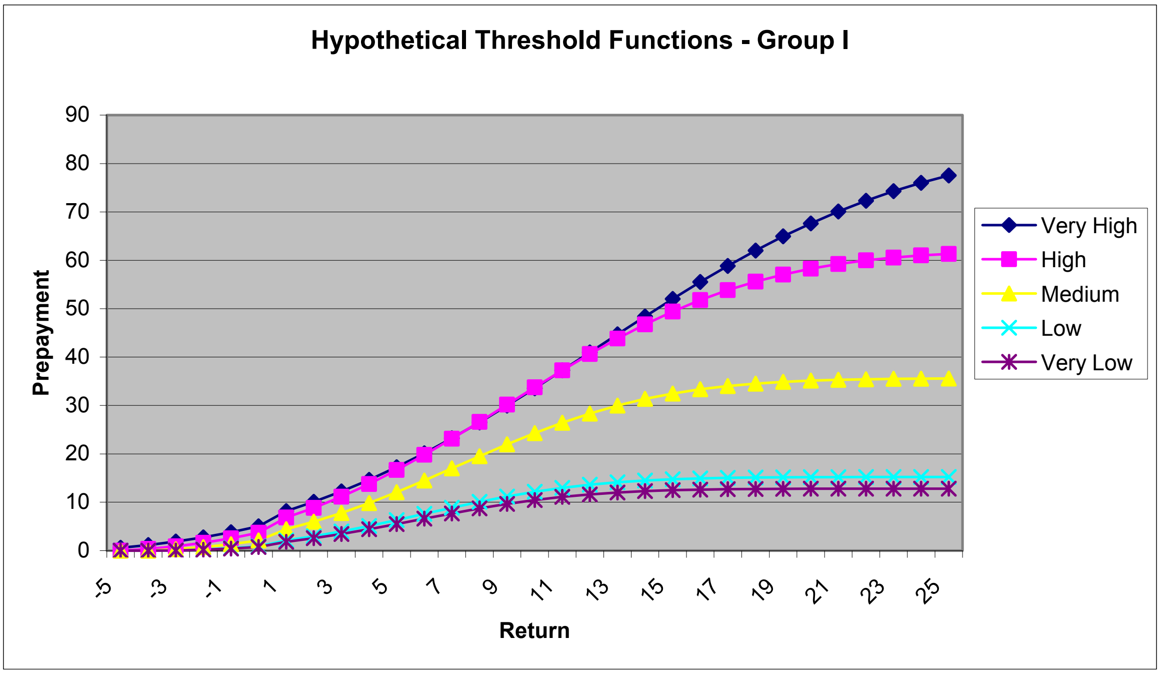
Taking these observations into account – the overall result is indeed very satisfactory.

[13] This formulation is closely related to the models from Dahl (1997) and Madsen (1998).
[14] The estimation method used is a subspace trust region method and is based on the interior-reflective Newton method described in Coleman and Li (1994, 1996). Each iteration involves the approximate solution of a large linear system using the method of preconditioned conjugate gradients (PCG).
[15] In the ideal world we would like to have a Slope equal to 1 (one).

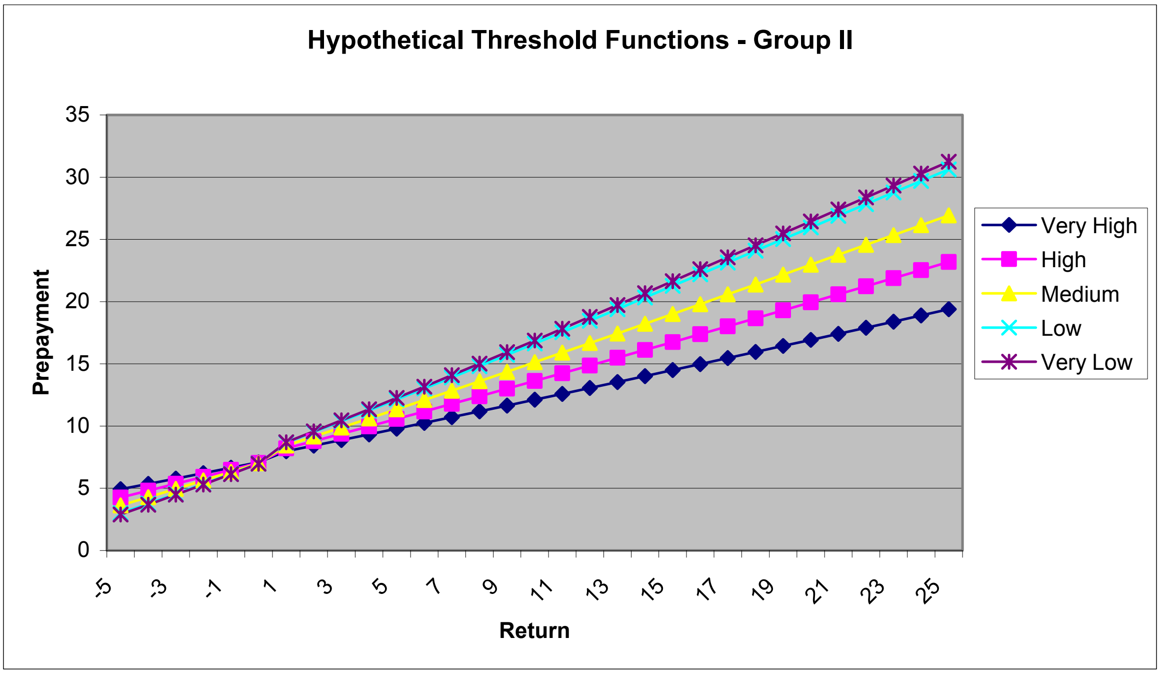
From an economic perspective it is very important that the estimated parameters can be given a reasonable and meaningful explanation. Here are our observations on the estimated parameter vector[16]:

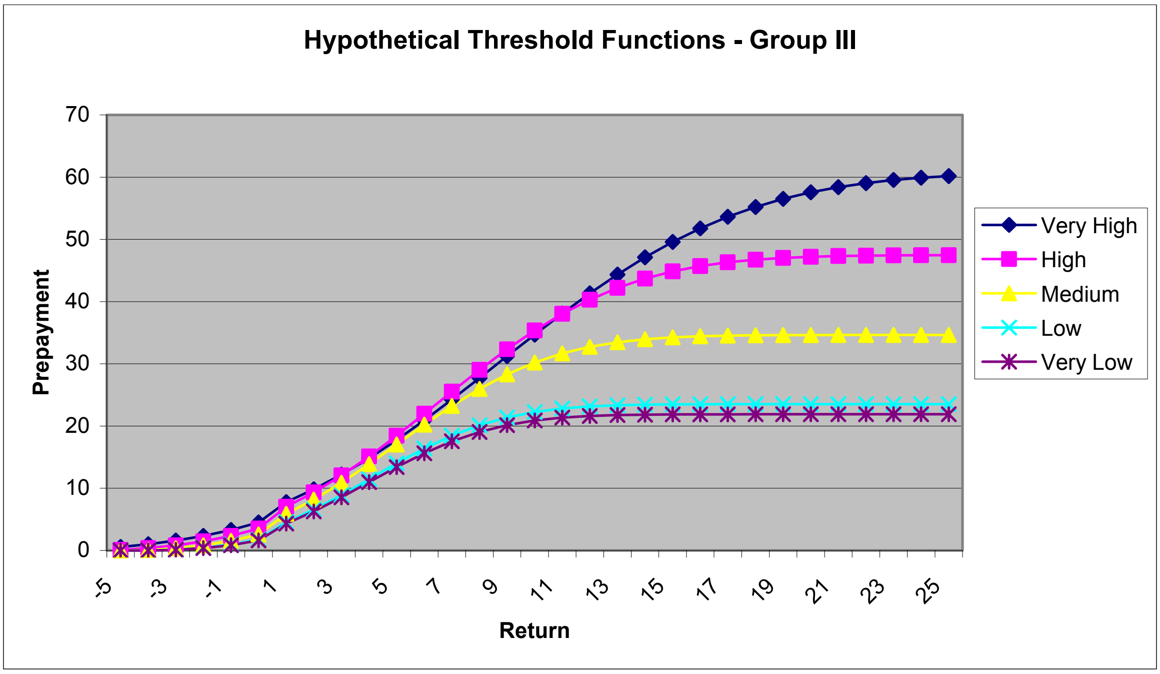
- Level: The estimated value is slightly negative. This seems reasonable given the fact that the model does not take into account prepayments due to an increase in the maturity of the loan
- YCSlope: The steepness of the yield-curve has a positive impact on prepayments[17]
- YCChange: The fall in the refinancing yields from one period to the next period has a positive impact on the prepayments
- Maturity Ratio[18]: The Maturity Ratio is positively related to the size of the prepayments
- PoolFactor: The dampening effect of the pool-factor on the size of the prepayments are non-linear[19] in nature

To illustrate the differences between the estimated values for the mean "Return" and standard deviation around the mean "Return", we have below shown the threshold function for each of the 3 groups for a selected range of pool-factors (Very High High, Medium, Low, Very Low)[20]:



[16] The points given here, is based on the estimation result on the total dataset, however the same conclusion applies to the results obtained on a subset of the total dataset.
[17] This can be termed the Flex-Loan effect.
[18] The maturity ratio is defined as: $\dfrac{Mat_{remaining}}{Mat_{\max}}$.
[19] This is for example supported by the result from Madsen (1998, 2002) and Nordea (2004).
[20] It is assumed that the maturity ratio is equal to 1.





One important point to emphasize is that the reason the word "Hypothetical" is used is due to the fact that there are some parts of the threshold functions that are to be considered impossible combinations. One example is the combinations of high returns and very high pool-factors. The reason for this is that returns do not jump! (At least not that much)

Still, there are some interesting observations that are worth mention here:

- For Group 1 and Group 3 we can observe that the prepayment is positively correlated with the pool-factor (a result we expect, because that is how we meant that the so called burnout effect would materialize)
- The decline in prepayments for Group 1 for falling pool-factors is much more severe than what can be observed for Group 3

- The prepayment for Group 2 is negatively correlated with the pool-factor for positive returns and positively correlated for negative returns
- For high and very high pool-factors the prepayment for Group 1 is higher (marginally for not large returns) than the prepayment for Group 3 – whereas the opposite holds true for medium, low and very low pool-factors
- For small returns the prepayment for Group 2 is higher than the prepayments for Group 1 and Group 3 – irrespectively of the pool-factors
- For medium, high and very high pool-factors and relative high returns the prepayment for Group 2 lies below the prepayments for Group 1 and Group 3

The above observations indicate a few surprises - let us highlight these unexpected features:

1. For relative high pool-factors and for low to medium high returns the prepayment speed for Group 1 and Group 2 are very similar
2. For Group 2 we have that the prepayment speed (for positive returns) rise with a decline in the pool-factors

If we take into account that the estimation period "only" covers 2 years of data it might not come as a surprise that some divergence from the overall structure can occur. A regime shift is probably a better way to put. The prepayment behavior of the house owners has been changing over time. Gradually over the last 10-15 years we have seen an enormous change in awareness – with today (as mentioned earlier) makes prepayment a kind of national sport! Still the 2 points mentioned above deserve some comments.

With respect to case 1 – then we believe this is a new feature for Danish mortgage bonds. The reason for this - at first glance strange observation - is as follows:

- Over the last 2-4 of years there has been an increase in other loan forms for the house owner, now bank loans (ARMs/variable APR[2]) are in much more use – mainly due to competitiveness and the higher degree of flexibility they offer. The target have here been "smaller size" loans. In general there has been an increased focus on "smaller size" loans – this can for example also be observed in the mortgage deed market, where the prepayments over the last couple of years have been historically high[21]

We believe this is the main explanation for case 1.

Case 2 can be understood as a snow ball effect i.e. for constant (positive) returns the prepayment will be increasing. It is another way to put that the bonds do not get burned out but continues to prepay "big time" every quarter. In the market this phenomena has been discussed and some market participants excluded the pool-factor from their model while others thought it was an indication for a new regime shift due to continuously falling rates and new products. Another way to think about it is as a kind of refueling[22]. Remember that Group 2 consists of neither small size loans nor large size loans (Group 2: 0.5-3 mill.). This Group is harder to classify because it is much more diversified than the 2 other Groups. Taking into account that the prepayment for any given mortgage bond is derived using equation 1, then the reason for case 2 can be formulated as follows:

- It is true that refueling (or some kind of snow ball effect) can be observed in the market. That this effect should be driven by Group 2 alone does not (at first glance) seem to be a valid conclusion. However, an observation similar to the one we see for Group 2 has to show up in the prepayment model (one way of the other) – if not, then refueling features (snow ball effects) will not be an integrated part of the model (as it should be). Thinking about it, having this feature dominating the most diversified group, namely Group 2 – does seem appropriate

All in all we believe that the estimated model has the right economic features incorporated, and we are also certain that the interaction between the 3 Groups will ensure an intuitive and valid relationship between the total prepayment speed, the level of returns, the pool-factors and the remaining time-to-maturity.

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[21] A good explanation for this is the fact that there has been a high increase in property value over the last 5-10 years, with makes it possible to convert the more expensive mortgage deed loans to traditional mortgage bond loans (or alternatively to a bank loan).
[22] For a discussion of the concept refueling see Madsen (2001).

The Par Rule

From a practical point of view, it is important to ensure that there is no prepayment in the model when the clean price of the mortgage bond is below par. There is however from a modeling point of view no way that we can make certain that this does not happen – we can at best hope to minimize the probability for this to happen. In FiPaM this is done by adjustment the prepayment-rate as follows:

(5)
$$\lambda_{new} = \lambda P(\Pr ice \geq 100)$$

That is the prepayment rate at a given state is being normalized by the probability that the clean price of the mortgage bond at a given state is above or equal to 100.

Some Closing Remarks

Prepayment modeling is not an exact science, it is a question of finding/selecting the relevant variables that best describes the observed behavior – with due respect to the fact that the model has to be economic valid.

We believe that the model specified here captures all the major and important ingredients for the prepayment behavior of Danish mortgage bonds. Furthermore, due to the fact that the model is economic viable, it also gives hope that it will behave well under changing market conditions.

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Literature

Coleman and Li (1994): ”On the convergence of Reflective Newton Methods for Large-Scale Nonlinear Minimization Subject to Bounds”, Mathematical Programming, Vol. 67, no. 2, page 189-224, 1994

Coleman and Li (1996): ”An Interior, Trust Region Approach for Nonlinear Minimization Subject to Bounds”, SIAM Journal of Optimization, Vol. 6, page 418-445, 1996

Dahl (1997): ”S-E-Bankens realkreditmodel I – debitoradfærd”, (in Danish) working paper SEB, 3. June 1997

Danske Research (2001): ”Danish Mortgage Bonds – A market description”, 14. November 2001

Madsen (1998): ”The Modelling of Debtor Behavior”, Svenska Handelsbanken, Financial Reference Library no. 14. March 1998

Madsen (2001): ”BG Banks Prepayment Model”, (in Danish) working paper BG Bank, 20. March 2001

Unibank (2004): ”Unibanks Realkredit Model”, (in Danish) Presentation Spring 2004