Barbell-Strategies - revisited

1: Introduction

The main purpose of this paper is to give a thorough explanation of three different Barbell strategies; Standard Barbells, Butterfly Barbells and Box Barbells. This paper extends the results in “Barbells - A Nordic Perspective” and “Barbells, Butterflies and Boxes - Three Barbell strategies”.

Apart from looking at how each strategy is defined and their structure we are focusing on the differences and relations that exist between the three Barbell strategies. Both in connection with the risk in each kind of Barbell and the pick-up implied from each strategy.

The main contribution of this paper is a reformulation of the Box Barbell strategy, a thorough discussion at how Barbell pick-ups should be calculated and a detalied analysis of the relationsship between the risk and return of different Barbell stategies. Furhermore we analyse the relationship between the pick-up implied from the three Barbell strategies and their connection to a decomposition of the yield changes.

In section 2 we will give a short description of how the weights on each wing in the three Barbell strategies should be computed and discuss the risk imposed by the each Barbell strategy. In section 3 an analysis of how the pick-up should be calculated is done and the relationship to a decomposition of yield changes is performed.

2: Barbell Strategies - a Short Description

As a Barbell strategy consists of selling/buying the middle[1] bond and buying/selling the wings, we need some kind of restrictions in order to be able to make a unique determination of the nominal weights for each of the legs. How these restrictions are defined is what destinguishes different Barbell strategies from each other. Put in another way it is the restrictions that precisely determine which kind of trade is being entered into. This will be explained in detail below.

In this paper we will discuss the following three kinds of Barbell strategies:

- Standard Barbells
- Butterfly Barbells
- Box Barbells

[1] The concept normally used in determining the relationship that allowa us to use the terminology left wing, middle bond and right wing. is basis point value or dollar duration. Remember that dollar duration is defined as k = P_y - that is as the first derivative of the price with respect to YTM (yield-to-maturity).

These three different kinds of Barbell strategies have been thoroughly discussed in the two Reference Libraries “Barbells - A Nordic Perspective” and “Barbells, Butterflies and Boxes - Three Barbell Strategies”. Because af this only a short description of these strategies and their relationship will be given here. When discussing Barbell strategies we will (if nothing else is mentioned) assume that we are short in the middle bond and long in the wings.

Before we will begin our short description of the three different Barbell strategies it is of importance to establish a framework which allows us to relate the yields of the bonds to each other.

We have chosen (as is usually the case) to decompose this relationship into three parts:

- The Level - which is represented by the yield on the left wing:
  $L = r_l$
- The Slope - which we define as the yield spread between the right and the left wing, normalized by the difference in modified duration (risk difference), i.e.
  $S = \dfrac{r_r - r_l}{D_r - D_l}$
- The Relative value of the middle bond (RVMB), which is defined as:
  $RVMB = L + S(D_m - D_l)$

This also means that when we use the concept yield-curve we are actually referring to the straight line between the yields on the left and right wing.

2.1 Standard Barbells

The restrictions used in this connection can mathematically be formulated as follows:

$$\begin{aligned} k_m &= xk_l + yk_r \\ P_m &= xP_l + yP_r \end{aligned} \tag{1}$$

Where k_m, k_l and k_r are respectively the dollar duration for the middle bond and the left and right wings. P_m, P_l and P_r are defined as the price (including accrued interest rate) on the middle bond and the left and right wings. Lastly x and y are defined as the nominal amounts for the two wings.

From formula 1 it follows that in the standard Barbell strategy we want to ensure that the new position has the same dollar duration as the existing position and furthermore the trade should be liquidity neutral.

Given formula 1 we find that in the Standard Barbell strategy the weights are given as:

$$x = \frac{k_m}{k_l} - \frac{k_r}{k_l}y \;\;;\;\; y = \frac{P_m k_l - P_l k_m}{P_r k_l - P_l k_r} \tag{2}$$

Usually we consider this strategy as being hedged against equal changes in the yields on the three bonds[2]. From this assumption it follows that the strategy is sensitive to non-parallel shifts in the yields on each of the three bonds.

This means that the strategy can be considered to be a curve-trade. By this we mean that if we are long in the wings the strategy will in general be profitable if we see an inversion of the yield-curve and the strategy will lose money if we see a steepening in the yield-curve.

Let us illustrate this by an example:

Table 1: Standard Barbell Strategy (Trading date: 27 April 1998)

	Left Wing	Middle Bond	Right Wing	
Bond	4% INK Stgb I 00	08.00 St.l n 03	7% INK St.lån 07	Combined Position
Clean Price	99.20	113.45	112.90	
Acc. Interest	0.83	-0.33	3.21	
Price	100.03	113.12	116.11	
Yield	4.47	4.92	5.24	
Modified Duration	1.68	4.19	6.86	
Dollar Duration	1.68	4.74	7.97	4.74
Position	58.24	-100.00	47.25	
Value/Cash-payout	58.26	113.12	54.86	0.00

Table 2: Return Simulation Matrix[3]

Left/Right Wing	**-1**	**-0.75**	**-0.5**	**-0.25**	**0**	**0.25**	**0.5**	**0.75**	**1**
-1.00	0.06	-0.36	-0.75	-1.13	-1.49	-1.83	-2.15	-2.46	-2.75
-0.75	0.45	0.03	-0.37	-0.75	-1.11	-1.46	-1.78	-2.1	-2.39
-0.50	0.84	0.42	0.01	-0.37	-0.74	-1.09	-1.42	-1.74	-2.04
-0.25	1.23	0.8	0.39	0	-0.37	-0.72	-1.06	-1.38	-1.68
0.00	1.61	1.18	0.77	0.38	0	-0.36	-0.7	-1.02	-1.33
0.25	1.99	1.56	1.14	0.74	0.37	0	-0.34	-0.67	-0.98
0.50	2.37	1.93	1.51	1.11	0.73	0.36	0.01	-0.32	-0.63
0.75	2.75	2.3	1.88	1.47	1.09	0.72	0.37	0.03	-0.29
1.00	3.12	2.67	2.24	1.84	1.44	1.07	0.71	0.37	0.05

[2] This concept is however more restrictive than is actually the case with this kind of strategy. The reason for this is that for a given change in yield on the middle bond there exist an infinite number of ways that the yields on the left and right wing are allowed to change while still preserving that the change in value is equal for the two positions.

[3] In this example we have performed a simultaneus change of the yields on the left and right wings. With respect to the yield on the middle bond we have assumed that it is approximately equal to RVMB (what we mean by that will be explained in section 3). The Level, Slope and adjusted RVMB used in this example are shown in Appendix A.

The return shown in table 2 is defined as the weighted[4] value of the wings minus the value of the middle bond[5] and the up-front cash payout.

The return associated with a parallel shift in the yields on all three bonds is represented by the diagonal in table 2. From here it can be seen that the strategy is hedged against this particular form for yield-dynamics. From the table it also follows that an inversion of the yield-curve (falling long-rates combined with rising short-rates) results in positive returns. A steepening of the yield-curve (rising long-rates combined with falling short-rates) - as mentioned above - gives negative profits.

From table 2 it is easily seen that for symmetric yield-curve changes the profit is always higher than the opposite loss - the reason for this is because the long bond (right wing) has higher convexity than the short bond (left wing).

2.2 Butterfly Barbells

The restrictions used in this connection can be formulated mathematically as follows:

$$\begin{gathered} k_m = xk_l + yk_r \\ xk_l = yk_r \end{gathered} \tag{3}$$

This strategy is designed to be dollar duration neutral and furthermore the dollar duration on each wing should coincide.

Given formula 3 we find that in the Butterfly Barbell strategy the weights are given as:

$$x = 0.5\frac{k_m}{k_l} \quad ; \quad y = 0.5\frac{k_m}{k_r} \tag{4}$$

As follows from the second criteria this strategy is generally considered to be a curve-neutral strategy. That is however not completely true, as it is only curve-neutral with respect to a particular kind of twist in the yield-curve.

From formula 3 it can be derived that the Butterfly strategy is curve-neutral with respect to equal changes in the yields on the left and right wing. The kinds of twist in the yield-curve that the strategy is hedged against are therefore yields changes of equal size but of opposite sign - with respect to the restriction that the yield change of the middle bond has to be 0[6].

Graphically this can be shown as:

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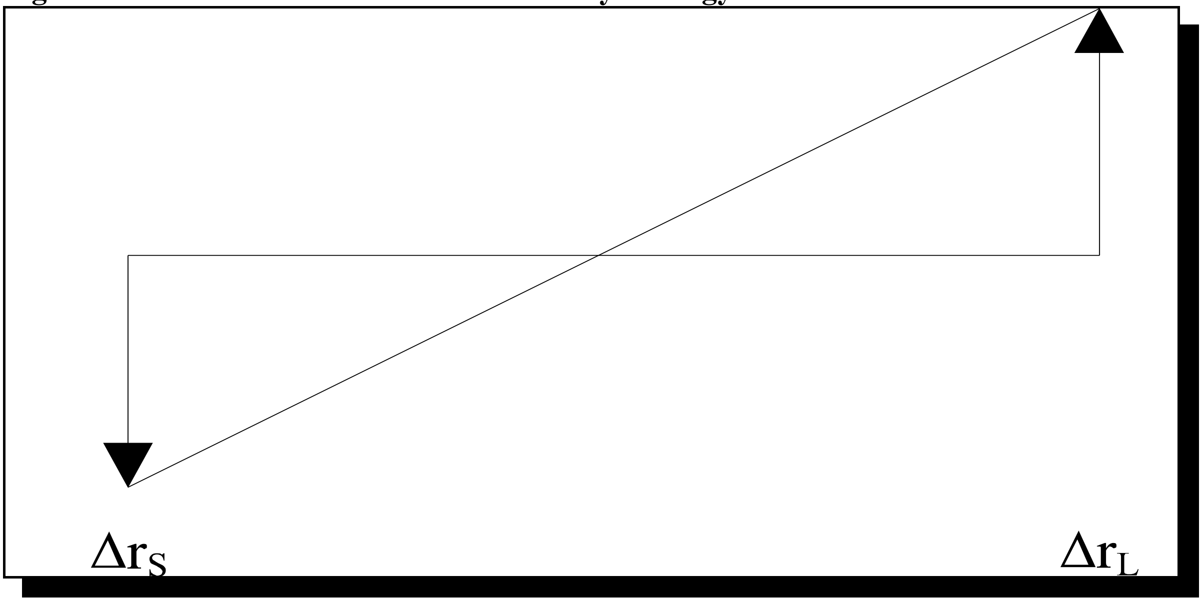
[4] Where the weights are given by the positions in table 1.

[5] The values calculated as a function of the assumed yield-dynamics are not calculated by re-evaluating each bond at the new yield level, but through a risk-measure approximation approach - se Appendix B for further details.

[6] Given the RVMB definition, the kind of twist the Butterfly Barbell takes into account can be relaxed a little, see section 2.3.

Figure 1: Twist assumed in the Butterfly strategy



From formula 3 it can be seen that the strategy is not designed to be liquidity neutral. The reason for this follows as a direct consequence of criteria 2 in formula 3: from that it follows that this particular form of curve-neutrality is established by buying more short bonds (left wing) and less long bonds (right wing).

In this connection it should be pointed out that it is generally thought that in order to make the position curve-neutral, always gives rise to an up-front initial cash-payout. This is however not generally the case for the kind of curve-neutrality which is implicitly assumed in the Butterfly Barbell. But for the general case of curve-neutrality (the Box Barbell strategy, see section 2.3) there will always be an up-front initial cash-payout.

Let us illustrate the Butterfly Barbell strategy by an example:

Table 3: Butterfly Barbell Strategy (Trading date: 27 April 1998)

	Left Wing	**Middle Bond**	**Right Wing**	
Bond	4% INK Stgb I 00	08.00 St.l n 03	7% INK St.lån 07	Combined Position
Clean Price	99.20	113.45	112.90	
Acc. Interest	0.83	-0.33	3.21	
Price	100.03	113.12	116.11	
Yield	4.47	4.92	5.24	
Modified Duration	1.68	4.19	6.86	
Dollar Duration	1.68	4.74	7.97	4.74
Position	141.28	-100.00	29.76	
Value/Cash-payout	141.32	113.12	34.55	-62.75

Table 4: Return Simulation Matrix

Left/Right Wing	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
-1.00	0.01	-0.03	-0.05	-0.07	-0.07	-0.07	-0.06	-0.04	-0.01
-0.75	0.05	0.01	-0.02	-0.04	-0.06	-0.06	-0.05	-0.03	0
-0.50	0.08	0.04	0	-0.02	-0.04	-0.04	-0.04	-0.02	0
-0.25	0.12	0.07	0.03	0	-0.02	-0.03	-0.03	-0.02	0
0.00	0.15	0.1	0.06	0.02	0	-0.01	-0.02	-0.01	0
0.25	0.19	0.13	0.08	0.04	0.02	0	-0.01	-0.01	0
0.50	0.22	0.16	0.11	0.07	0.03	0.01	0	0	0.01
0.75	0.25	0.19	0.13	0.09	0.05	0.03	0.01	0	0.01
1.00	0.29	0.22	0.16	0.11	0.07	0.04	0.02	0.01	0.01

From table 4 it is obvious that this kind of strategy in comparison with the Standard Barbell strategy is approximately curve-neutral.

2.3 Box Barbells

A more general kind of curve-neutrality than the Butterfly strategy allows for can be obtained in the following way:

We now want to define a strategy which satisfies the following general relationship:

$$k_m \Delta r_m = x k_l \Delta r_l + y k_r \Delta r_r \qquad \textbf{(5)}$$

In order to be able to get a unique solution for x and y we need to specify a relationship between yields. If we now assume that RVMB is a proxy for the yield on the middle bond, ie.

$$r_m = r_l + \frac{D_m - D_l}{D_r - D_l}(r_r - r_l) \qquad \textbf{(6)}$$

Then formula 5 can be rewritten as:

$$k_m \Delta r_l + k_m \frac{D_m - D_l}{D_r - D_l}(\Delta r_r - \Delta r_l) = x k_l \Delta r_l + y k_r \Delta r_r \qquad \textbf{(7)}$$

Re-arranging terms in formula 7 in order to separate the yield shift associated with the short and long bonds respectively, gives:

$$\Delta r_l \left(k_m - k_m \frac{D_m - D_l}{D_r - D_l} - x k_l \right) = \Delta r_r \left(y k_r - k_m \frac{D_m - D_l}{D_r - D_l} \right) \qquad \textbf{(8)}$$

As the left hand side needs to be equal to the right hand side of equation 8, allows us to conclude that the weights for each leg in the Box Barbell strategy is defined as:

$$x = \frac{k_m}{k_l}\left(1 - \frac{D_m - D_l}{D_r - D_l}\right) \quad ; \quad y = \frac{k_m}{k_r}\left(\frac{D_m - D_l}{D_r - D_l}\right) \qquad \textbf{(9)}$$

In the Box-Barbell strategy the weights on each leg are determined as specified in formula 9. As is obvious from the derivation this strategy is - given the yield relationship decomposition - non-sensitive to all kind of yield movements - if we disregard higher order moments (such as convexity).

If we look at how the weights are determined in the Butterfly Barbell and compare with formula 9 it is easily seen that the Box Barbell strategy will degenerate to the Butterfly Barbell if the modified duration of the middle bond is defined as $0.5\left(D_l + D_r\right)$, i.e lies - in a risk perspective - exactly between the left and right wing.

Using this knowledge, it follows that the Butterfly Barbell and the Box Barbell, for the Barbell strategy considered in for example section 2.2 will behave almost identical. The difference between the two strategies is therefore better explained by choosing another strategy.

Let us illustrate this by an example[7]:

Table 5: Butterfly Barbell Strategy (Trading date: 27 April 1998)

	Left Wing	**Middle Bond**	**Right Wing**	
Bond	4% INK Stgb I 00	04.00 INK Stgb I 01	7% INK St.lån	Combined Position
Clean Price	99.20	98.33	112.90	
Acc. Interest	0.83	0.83	3.21	
Price	100.03	99.16	116.11	
Yield	4.47	4.65	5.24	
Modified Duration	1.68	2.56	6.86	
Dollar Duration	1.68	2.54	7.97	2.54
Position	75.55	-100.00	15.91	
Value/Cash-payout	75.57	99.16	18.47	5.11

Table 6: Box Barbell Strategy (Trading date: 27 April 1998)

	Left Wing	**Middle Bond**	**Right Wing**	
Bond	4% INK Stgb I 00	04.00 INK Stgb I 01	7% INK St.lån	Combined Position
Clean Price	99.20	98.33	112.90	
Acc. Interest	0.83	0.83	3.21	
Price	100.03	99.16	116.11	
Yield	4.47	4.65	5.24	
Modified Duration	1.68	2.56	6.86	
Dollar Duration	1.68	2.54	7.97	2.54
Position	125.47	-100.00	5.40	
Value/Cash-payout	125.51	99.16	6.27	-32.62

Table 7: Return Simulation Matrix (Buttefly Barbell Strategy)

Left/Right Wing	**-1**	**-0.75**	**-0.5**	**-0.25**	**0**	**0.25**	**0.5**	**0.75**	**1**
-1.00	0.03	-0.2	-0.43	-0.64	-0.85	-1.06	-1.25	-1.44	-1.62

[7] The Level, Slope and adjusted RVMB for this strategy are shown in Appendix C.

-0.75	0.25	0.02	-0.21	-0.43	-0.64	-0.84	-1.04	-1.23	-1.41
-0.50	0.47	0.23	0.01	-0.21	-0.42	-0.63	-0.82	-1.02	-1.2
-0.25	0.68	0.45	0.22	0	-0.21	-0.42	-0.61	-0.81	-0.99
0.00	0.9	0.66	0.43	0.21	0	-0.21	-0.41	-0.6	-0.78
0.25	1.11	0.87	0.64	0.42	0.21	0	-0.2	-0.39	-0.58
0.50	1.32	1.08	0.85	0.63	0.42	0.21	0.01	-0.19	-0.37
0.75	1.53	1.29	1.06	0.84	0.62	0.41	0.21	0.02	-0.17
1.00	1.73	1.49	1.26	1.04	0.82	0.61	0.41	0.22	0.03

Table 8: Return Simulation Matrix (Box Barbell Strategy)

Left/Right Wing	**-1**	**-0.75**	**-0.5**	**-0.25**	**0**	**0.25**	**0.5**	**0.75**	**1**
-1.00	0	0	-0.01	-0.01	0	0	0.01	0.02	0.03
-0.75	0.01	0	0	0	0	0	0.01	0.02	0.03
-0.50	0.01	0	0	0	0	0	0.01	0.01	0.02
-0.25	0.02	0.01	0	0	0	0	0.01	0.01	0.02
0.00	0.02	0.01	0	0	0	0	0	0.01	0.02
0.25	0.02	0.01	0.01	0	0	0	0	0.01	0.01
0.50	0.02	0.01	0.01	0	0	0	0	0	0.01
0.75	0.03	0.02	0.01	0	0	0	0	0	0.01
1.00	0.03	0.02	0.01	0	0	-0.01	-0.01	0	0

From table 7 it can be deduced that for $D_m << 0.5(D_l + D_r)$ the Butterfly Barbell strategy behaves more like a Standard-Barbell strategy than a curve-neutral strategy[8].

It could be argued that this conclusion is a bit unfair for the Butterfly Barbell strategy - as yield-changes in the middle bond are implied from RVMB - and the Box Barbell is derived to be consistent with this relationship. The point to make here is that implying the yield on the middle bond from RVMB is the only reasonable way when a simulation is done in order to analyse how the strategy behaves under different yields.

Of course we could restrict ourselves to parallel shift in the yields on all three bonds, and pivoting of the yield-curve around the initial level of yield on the middle bond[9].

An obvious question to ask at this point is - how could it then be that for the Butterfly Barbell strategy in section 2.2 the Return Simulation Matrix in table 4 had results which were in line with what we expected even though the yield of the middle bond was implied from RVMB?

[8] This is however not a general conclusion. In the case when $D_m << 0.5(D_r + D_l)$ (which is the case in the above example) the Butterfly Barbell will behave like a Standard Barbell. But in the case when $D_m >> 0.5(D_r + D_l)$ the Butterfly Barbell will behave like a Standard Barbell where you are short in the wings - this case is shown in Appendix D.

[9] This kind of simulation is actually the only one considered by Bloomberg.

The reason for what is at first sight an inconsistency between the results reported in the above example for the Butterfly Barbell compared to the results obtained in section 2.2 is: When the modified duration of the middle bond is approximately equal to $0.5(D_r + D_l)$ then the yield implied from RVMB will be consistent with the yield-changes that the Butterfly Barbell takes into account.

3: Pick-Up calculation - a discussion of methods

Nothing so far has been said about how to determine the yield pick-up - with of cource is of great importance.

In general we have that the yield pick-up is defined as $\rho = r_b - r_m$, where ρ is the pick-up, r_b is the yield on the portfolio consisting of the two wings and r_m is the yield on the middle bond.

From this it can be deduced that the pick-up calculation method is a direct function of how we decide to measure the yield on the portfolio.

Usually two different ways to compute the yield on the portfolio are considered:

$$SWY = \frac{xP_l r_l + yP_r r_r}{xP_l + yP_r}$$

$$DWY = \frac{xE_l r_l + yE_r r_r}{xE_l + yE_r} \qquad \textbf{(10)}$$

E in the formula represents the elasticity of the bond.

The relationship between E and D is: $E = D(1 + r)$.

It can be shown that DWY is an analytical approximation of the IRR on the portfolio, see Appendix E.

The yield on a bond is associated with a naive yield-dynamics (a constant yield assumption), i.e the Rate-Of-Return (ROR) is equal to the bond yield over a fixed period assuming no payments and a constant yield[10].

Let us illustrate this with an example:

Table 9: Return calculation for 8% St. Lån 03

	Primo date: 27 April 1998	Ultimo Date: 27 July 1998	ROR
Clean Price	113.45	112.82	

[10] If payments occur during the period it is assumed that they are reinvested to the IRR (yield).

Acc. Interest	-0.33	1.67	
Price	113.12	114.49	1.37
Yield	4.92	4.92	
ROR			4.92[11]

Unfortunately this relationship is not true if we consider a portfolio of two or more bonds. By this we mean that the ROR is not equal to the IRR of the portfolio. Put in another way DWY is not consistent with a naive yield-dynamics assumption. Actually we have that ROR for a portfolio is approximately given by SWY - that is a value-weighted yield on a portfolio is consistent with a naive yield-dynamics.

Let us illustrate this with an example[12]:

Table 10: Return calculation - portfolio (4% INK Stgb I 00 and 7% INK St. Lån 07)

	Primo date: 27 April 1998	**Ultimo Date: 27 July 1998**	**ROR**
Portfolio Value	113.12	114.46[13]	1.34
SWY Yield	4.84		
DWY Yield	5.06		
ROR			4.84

Why does not DWY work? After all ROR for a single bond equals its IRR = DWY.

Instead of showing that DWY is not consistent with a naive yield-dynamics I will show that SWY is[14].

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[11] ROR is calculated as: $ROR = 100\left(\left(\frac{P_u}{P_p}\right)^{\frac{1}{d}} - 1\right)$, where P_p and P_u are respectively the primo and ultimo price of the bond and d is the number of days (bond convention) divided by 360.

[12] We have used the positions from table 1 in the portfolio construction and the yields on the two bonds in table 1 in the calculation of the portfolio value.

[13] The horizon value has been calculated by assuming that the yields on each of the bonds in the portfolio stay constant.

[14] If we want to use the opposite argumentation - that is explain why DWY does not work, then this can be explained as follows: Even though the yields stays constant the weights in the formulæ for DWY and SWY will change as time goes. This is however irrelevant for SWY, as the change in the weights is not relevant in the calculation of ROR - we only need the beginning-of-period SWY. ROR can of course also be calculated using DWY,

Remember that ROR is defined as: $ROR = 100\left(\left(\frac{P_u}{P_p}\right)^{\frac{1}{d}} - 1\right)$, we also have that the relationship between P_p, P_u and the yield (r) on the bond at the primo date is given by the following equation[15]: $P_u = P_p(1 + r)^d$. Putting these two definitions together leads to ROR = r.

This observation can easily be used in the case of a portfolio of bonds to show that SWY is a good approximation for ROR - actually we see that for d = 1, SWY is exactly equal to ROR.

For the geneal case ($d \neq 1$) we can easily show that SWY is the right approximation for ROR. This can be done because the following relationship approximately exist: $(1 + r)^d \approx (1 + dr)$ - with this knowledge it follows immidiately[16] that SWY ≈ ROR.

This means that if we wish to use a naive yield-dynamics when we are calculating Barbell pick-ups it is necessary to use SWY as a proxy for the yield on the portfolio consisting of the left and right wing. We will therefore suggest that SWY is used in connection with the calculation of Barbell pick-ups[17].

In section 2 we have pointed out that we used an adjusted RVMB in the calculation of the sensitivities. The reason for this is that RVMB is **not** equal to the yield on the middle bond. What we actually have is a strict relationship between the yield on the middle bond, RVMB and the pick-up of a particulary Barbell strategy.

Proposition 1:

We assume the following relationship:

$$r_l + (r_r - r_l)\frac{D_m - D_l}{D_r - D_l} = \frac{xP_l r_l + yP_r r_r}{xP_l + yP_r} \qquad \textbf{(11)}$$

Where x and y are given as the solution to the Standard-Barbell strategy. That is the yield on the middle bond is equal to RVMB adjusted for the pick-up on the Standard Barbell strategy, ie $r_m = RVMB - (SWY_s - r_m)$.

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but then we need both the beginning-of-period and end-of-period DWYs - so in the DWY case the change in weights under a naive yield-dynamics is relevant/matters.

15 Under the assumption that no coupon payments occur over the period and that the nominal amount at the primo date is equal to the nominal amount at the ultimo date.

16 The proff is left for the reader.

17 When using the terminology Barbell pick-up we will for that reason refer to the case when the yield of the portfolio is calculated by using the SWY method.

Proof:

Plucking in the values for x and y given in formula 2 and re-arranging terms gives:

$$P_r(D_m - D_l) - P_r(D_r - D_l) = P_l(D_m - D_l)\left(\frac{k_r}{k_l} - \frac{k_m(P_r k_l - P_l k_r)}{k_l(P_m k_l - P_l k_m)}\right) \quad \textbf{(12)}$$

Re-writing it in terms of modified duration yields:

$$(D_m - D_l) - (D_r - D_l) = (D_m - D_l)\left(\frac{D_r}{D_l} - \frac{D_m(D_l - D_r)}{D_l(D_l - D_m)}\right) \quad \textbf{(13)}$$

Qed.

The implication from assumption 1 is furthermore that the pick-up in a Standard Barbell strategy will not change as a function of the simulation procedure used in section 2. We can also deduce that the pick-up in the Standard Barbell strategy will change according to the following equation:

$$\Delta\rho = \Delta r_l + (\Delta r_r - \Delta r_l)\frac{D_m - D_l}{D_r - D_l} - \Delta r_m^* \quad \textbf{(14)}$$

Where Δr_m^* is the actual change in the yield of the middle bond. From formula 14 it can be seen that the Box-Barbell is a hedge for the value on the combined position consisting of short the middle bond and long the wings given that the weight on the wings has been derived from the Standard-Barbell - or put another way - the Box-Barbell is an ideal tool for taking views of the development of the pick-up on the Standard-Barbell.

An interesting question to ask now is - how does the pick-up for respectively the Butterfly Barbell and Box Barbell strategy change as a function of the simulation procedure?

Before answering that question, it is of interest to mention that calculating the SWY or DWY in the case of positive or negative cash-payouts implies the following:

- If positive cash payout - it is assumed that the money is invested at a yield equal to the yield on the portfolio
- If negative cash-payout - it is assumed that the money is being borrowed at a yield equal to the yield on the portfolio

In general for both the Butterfly and the Box strategy we will encounter negative cash payouts. The implication for this is that in a rising yield-curve environment, the rule will be that the actual borrowing rate will be lower than the portfolio yield - the implication is that we are underestimating the pick-up[18].

Because of the non-linear relationship between yield changes and price changes we have the

[18] The opposite argument can be constructed in the case of a falling yield curve environment.

following general result for both the Butterfly Barbell and Box Barbell strategy:

- If we have equal changes in the yields on the left and right wing then the pick-up stays constant (the parallel case)
- If we have rising yields on the left wing and falling yields on the right wing we will in general see an increase in pick-up (the twist case)
- If we have falling yield on the left wing and rising yield on the right wing we will in general see a fall in the pick-up (the steepness case)

Instead of going through tedious formulas showing this - let us instead explain the reason for it. In general the change in pick-up as a function of the simulation procedure is given by:

$$\Delta \rho = \Delta r_l + (\Delta r_r - \Delta r_l)\frac{D_m - D_l}{D_r - D_l} + SWY - SWY^* \qquad \textbf{(15)}$$

Where SWY * represents the portfolio yield after a change in the yields on the left and right wing. This formula just states that the change in pick-up is given by the change in the yield on the middle bond (represented by the first two parts on the right side of the equation sign) adjusted for the change in the portfolio yield.

For the purpose of explaining how the pick-up changes as a function of the simulation procedure let us recall the formula for SWY:

$$SWY = \frac{xP_l r_l + yP_r r_r}{xP_l + yP_r} \qquad \textbf{(16)}$$

Keeping r_l constant and further assuming that the change in P_r is larger than the associated yield change (i.e. change in r_r) we can deduce the following:

- If we see rising yields on the right wing then the denominator will rise relatively quicker than the numerator - resulting in a rise in pick-up - all else being equal
- If we see fallen yields on the right wing then the denominator will fall relatively qiucker than the numerator - resulting in a fall in pick-up - all else being equal

The same kind of argument applies of course to the left wing[19]. Putting these arguments together gave rise to the general conclusions about the change in pick-up on Butterfly Barbells and Box Barbells as a function of the simulation procedure.

In general for all three kinds of Barbell strategies the change in pick-up is given by:

$$\Delta \rho = \Delta r_m^* + SWY - SWY^* \qquad \textbf{(17)}$$

Which for the Standard Barbell strategy degenerates to the relationship in formula 14.

An interesting thing to do now is to look at how the pick-up for all the three different Barbell

[19] In the case of one of the wings having a duration less than a year - the argument will of course be opposite - as the price change now will be less than yield change.

strategies is related to our yield-shift depomposition, when using actual yield-changes from the market.

For that purpose we have calculated the Level-factor, the Slope-factor and RVMB adjusted for the yield on the middle bond for a particular strategy - namely:

Middle Bond	**08.00 St. Lån 03**
Left Wing	**4% INK Stgb I 00**
Right Wing	**7% INK St. Lån 07**

We have chosen the period 27 October 1997 to 27 April 1998 as our reference trading dates.

The results are shown in the 3 graphs below:

Figure 2:

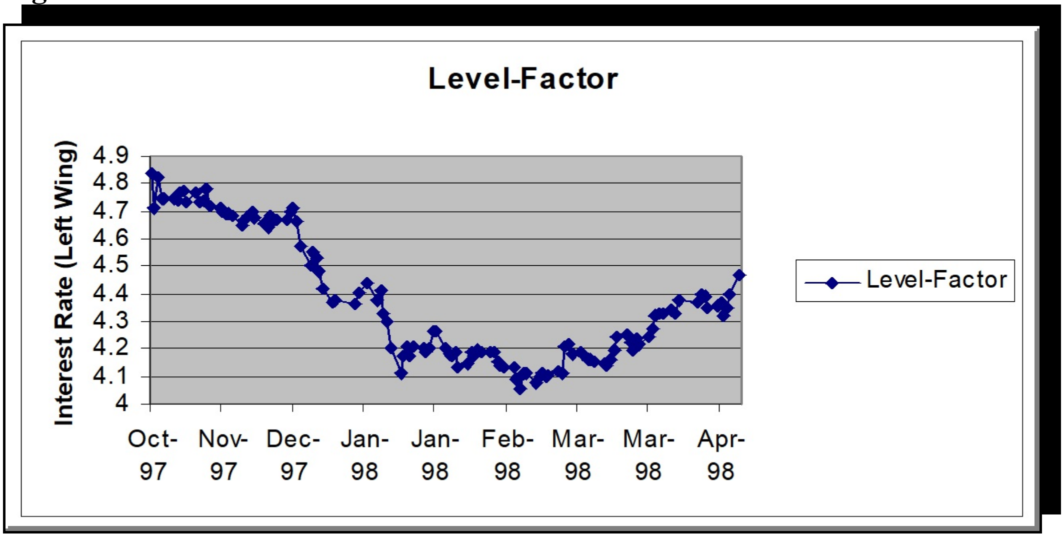


Figure 3:

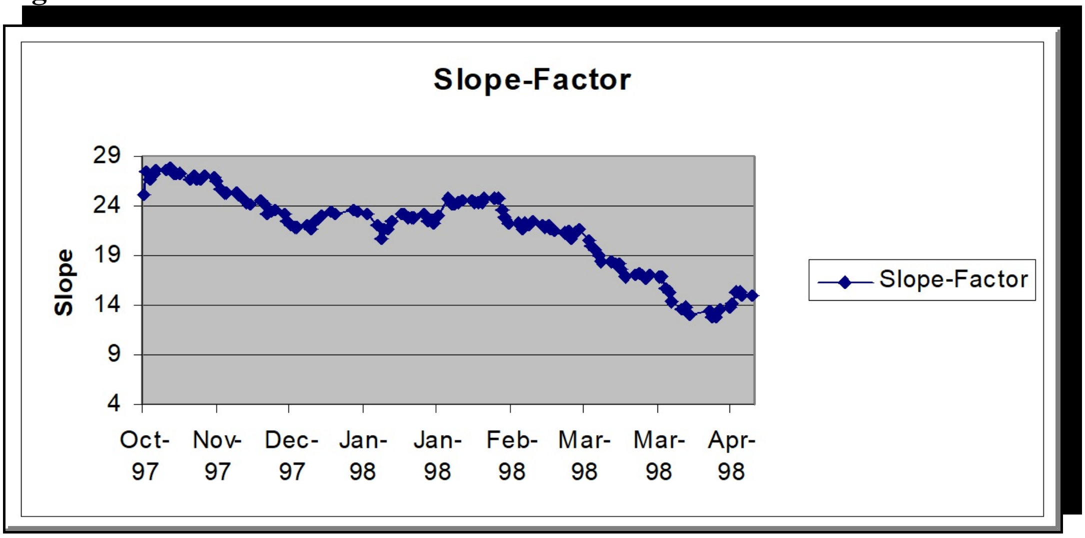
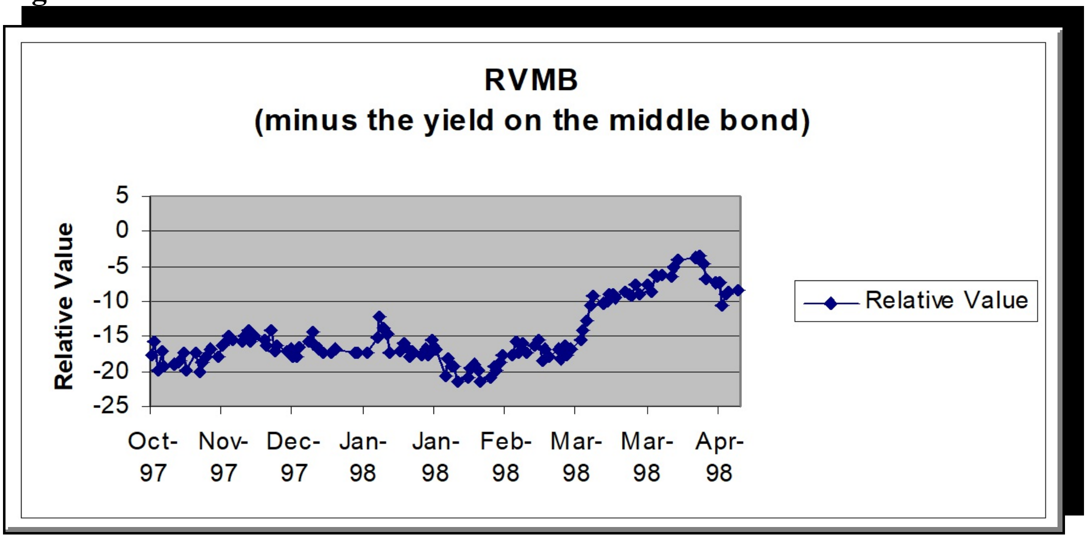


Figure 4:



For the three different kind of Barbell strategies we have calculated the SWY pick-up using two different approaches:

- The Fixed-Hedge pick-up: We have assumed that the nominal weights are constant for the whole period and have then calculated the evolution of the pick-up given the observed historical yields on the three bonds. The nominal weights are calculated for the last trading date, that is: 27 April 1998
- The Rolling-Hedge pick-up:: We have re-calculated the nominal weights at each trading-date and calculated the pick-up that is implied from this

From this we can deduce that for the trading date 27 April 1998 the two calculation methods must coincide.

A few other things might be worth mentioning:

- **Standard Barbells**
  - The Fixed-Hedge pick-up calculation method must result in a pattern that is closely related to the Relative Value
  - The Rolling-Hedge pick-up calculation method must be identical to the Relative Value (see Proposition 1)
- **Butterfly Barbells**
  - Both the Fixed-Hedge pick-up calculation method and the Rolling-Hedge calculation method must look quite similar to the Relative Value
- **Box Barbells**
  - Both the Fixed-Hedge pick-up calculation method and the Rolling-Hedge calculation method must look quite similar to the Relative Value
  - Compared to the Butterfly Barbell we must expect the difference in pick-up for the two different pick-up calculation methods must be less

than in the Butterfly Barbell case. The reason for this is that the Box Barbell is a more robust curve-hedge than the Butterfly Barbell

The two different pick-up calculation methods for the three different Barbell strategies are shown graphically below.

Figure 5

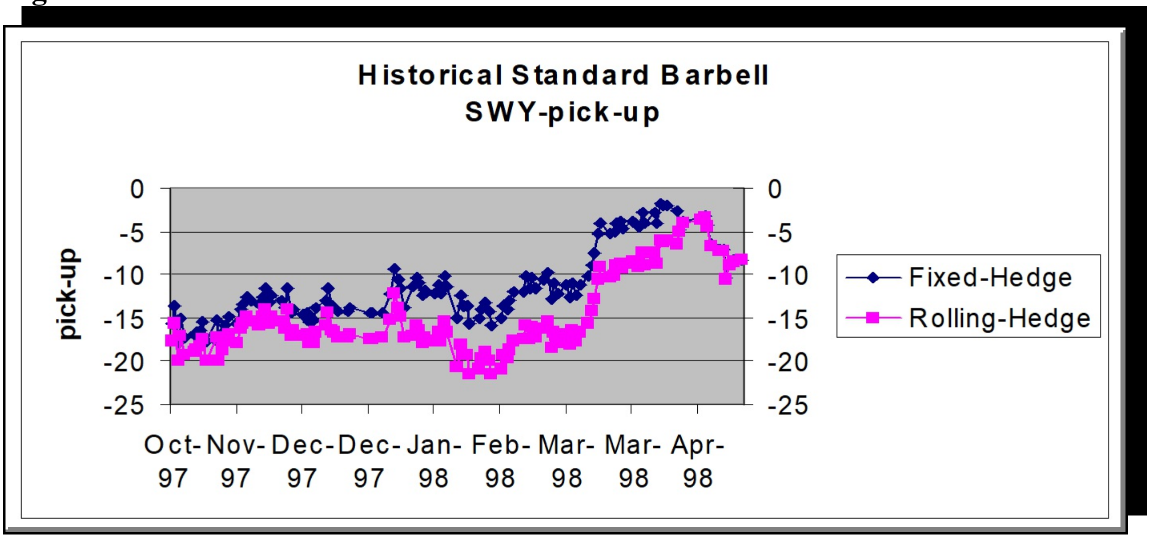


Figure 6:

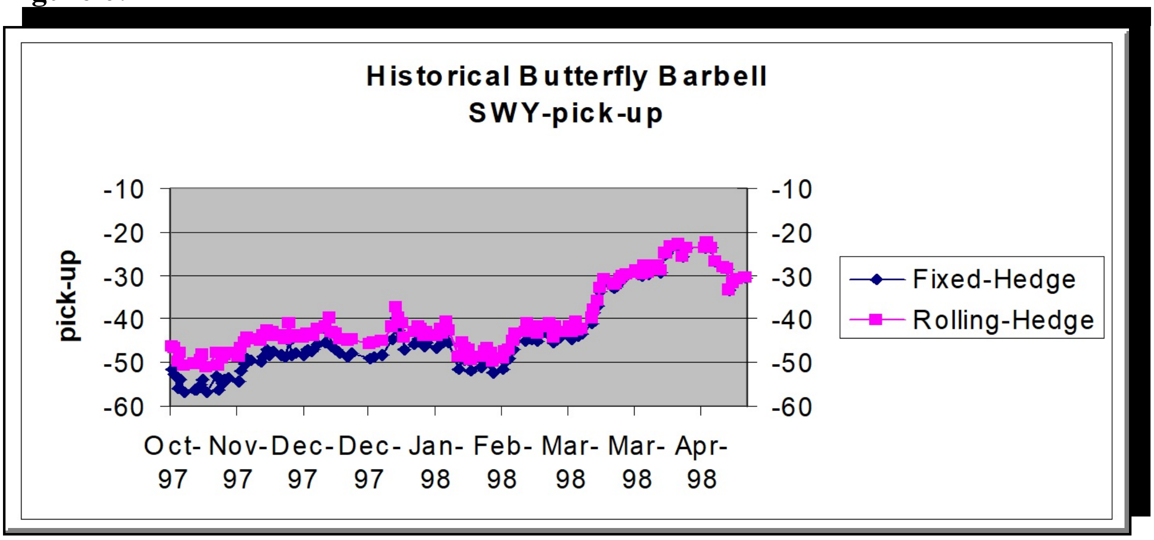
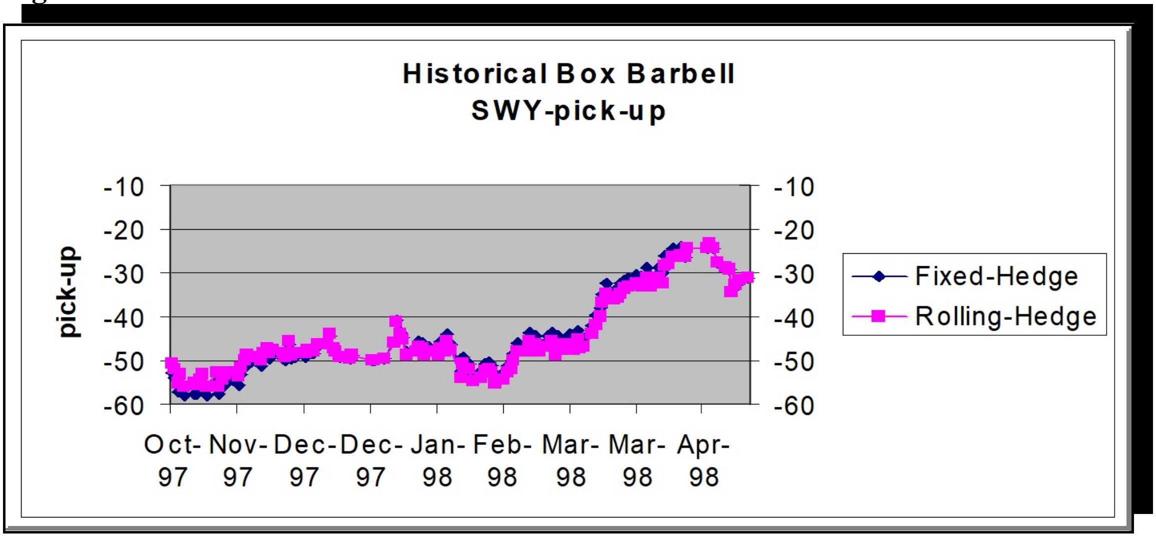


Figure7:



The observations regarding the relationship between the pick-up calculated using either the Fixed-Hedge approach or the Rolling-Hedge approach and the Slope Factor and Relative Value respectively is not independent of the pick-up calculation principle. What we mean by that is that these results are only true for the case of SWY calculated pick-ups.

The historical pick-ups using the DWY technique for the same strategy, same time period and for both the Fixed-Hedge and Rolling-Hedge implied pick-ups are graphically shown in Appendix F.

Referring to Appendix F let us point out the differences:

- **Standard Barbells**
  - The Fixed-Hedge pick-up calculation method and the Rolling-Hedge calculation method result in a pattern that is closely related to the Slope Factor

For both the Butterfly Barbell and the Box Barbell the relationship does not change as a function of the change in pick-up calculation principle.

The reason for this difference in interpretation is: As a Standard Barbell in a sense is a curve-trade a direct consequence of using the DWY approach is that the curve-sensitivity (represented by the Slope Factor) will be the main driving force in the evolution of the pick-up.

In the analysis done so far we have shown the following three important results:

- The SWY pick-up for the Standard-Barbell can be derived from our yield-curve decompostion (see Proposition 1)
- The Box-Barbell is (by construction) “value-neutral” with respect to the yield-curve decompositon
- The SWY pick-up is the only pick-up calculation method that is consistent

with a naive yield dynamics assumption

The implication from result 1 and 2 is the following: The Box-Barbell is a hedge[20] for the pick-up in the Standard-Barbell - or put another way (as mentioned earlier in this section) - the Box-Barbell is an ideal tool for taking views of the development of the pick-up on the Standard-Barbell.

This means that if we enter a Box-Barbell strategy we have established a hedge for the pick-up on the Standard-Barbell - but we have traded the Box-Barbell pick-up.

We have in Proposition 1 (together with the results from section 2.3) shown that the Box-Barbell is the strategy that hedge the SWY pick-up on the Standard-Barbell - the question is - can we be even more general than that? Is is for example possibly to deduce with Barbell strategy to trade if we want to hedge the SWY pick-up in the Box-Barbell or the Butterfly-Barbell?

The answer to this question is yes[21].

Before answering the question let me first generalise our yield-curve decomposition - and relate is to that particular form of Barbell strategy that is constructed to be "curve-neutral" with respect to this generalized yield-curve decomposition. Let us call this an α-Barbell.

Definition 1:

An α-Barbell is defined as a Barbell strategy that is non-sensitive to all kind of α-yield movements - if we disregard higher order moments (such as convexity).

Where an α-yield movement is defined through the following relationship between r_m, r_l and r_r:

$$\Delta r_m = \Delta r_l + \alpha\left(\Delta r_r - \Delta r_l\right) \qquad \textbf{(18)}$$

From definition 1 we can deduce that an α-neutral Barbell strategy is established if the weights x and y are defined as (see formula 9):

$$x = \frac{k_m}{k_l}(1-\alpha) \quad ; \quad y = \frac{k_m}{k_r}\alpha \qquad \textbf{(19)}$$

Proposition 2:

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[20] Hedge in this concept means the following: The combined position of being short the middle bond and long the Barbell is "only" a function of the change in pick-up. More precisely - is only a function of the difference between the actual yield change on the middle bond and the yield change on the middle bond derived from the yield decomposition.

[21] The following has been inspired by conversation with Erik Valtonen.

We now assume that the α-Barbell that has to be established in order to hedge the SWY pick-up of a particular Barbell strategy is defined as:

$$\alpha = \frac{yP_r}{xP_l + yP_r} \tag{20}$$

Proof:

From Proposition 1 and Definition 1 we have:

$$r_l + \alpha\left(r_r - r_l\right) = \frac{xP_l r_l + yP_r r_r}{xP_l + yP_r} \tag{21}$$

Re-arranging terms proves Proposition 2.

Qed.

Because of the result from Proposition 2 we can formulate the following general conclusion: When we enter into a Barbell strategy we trade the SWY pick-up for this particular strategy, but at the same time we are indirectly entering into a trade for taking views of the development of the SWY pick-up on a different Barbell strategy.

From equation 20 we can derive with kind of α-Barbell we would have to trade in order to hedge the SWY pick-up from a particular Barbell strategy. If we for example pluck in the values for x and y for the Standard-Barbell strategy we will find that $\alpha = \frac{D_m - D_l}{D_r - D_l}$ - which of course is exactly identical to the result from Proposition 1.

One could also find the α-Barbell that can be used to hedge the SWY pick-up of Box-Barbells and Butterfly-Barbells by using formula 20. That is however, not intuitively very interesting, after all, it is the geometrically intuitive definition that makes the SWY pick-up of the Standard-Barbell so appealing.

It is of course unfortunately that by entering a particular Barbell strategy we are only able to hedge the SWY pick-up for that particular Barbell strategy for parallel yield-curve changes. This actually implies that even though SWY works fine under a naive yield-dynamics assumption it does not behave well under in a dynamic yield-dynamics enviroment.

This observation actually leads to a rather surprising result - namely that under a dynamic yield-curve enviroment DWY makes more sense.

Proposition 3:

α-yield movements is equal to the DWY pick-up of an α-Barbell.

Proof:

From Definition 1 we have that the yield-curve dynamics for α-yield movements is defined as:

$$\Delta r_m = \Delta r_l + \alpha\left(\Delta r_r - \Delta r_l\right) \tag{22}$$

From formula 19 we have that the weights in a α-Barbell is defined as:

$$x = \frac{k_m}{k_l}(1-\alpha) \quad ; \quad y = \frac{k_m}{k_r}\alpha \tag{23}$$

The DWY pick-up is defined as:

$$\rho = DWY - r_m \tag{24}$$

Remember now that DWY[22] is defined as (see formula 10):

$$DWY = \frac{xk_l r_l + yk_r r_r}{xk_l + yk_r} \tag{25}$$

Putting into formula 25 the definition of x and y from equation 24 leads to the following result:

$$DWY = r_l + \alpha\left(r_r - r_l\right) \tag{26}$$

That is an α-Barbell can be used to trade its own DWY pick-up.

Qed.

We are now able to conclude the following:

- Under a naive yield-dynamics assumption SWY pick-up is the proper measure
- Under a dynamic yield-dynamics assumption DWY pick-up is the proper measure

Referring to Appendix F and comparing the pick-ups implied from using the DWY approach to the pick-up in figure 5-7 where we used the SWY approach - leads to the following important observation[23]:

It is easily seen that the pick-up derived from a DWY aproach is higher than the one derived from the SWY approach. This clearly gives a misleading picture of how the actual trade will behave under a naive yield-dynamics - it actually overestimates the pick-up. It is also seen that it is possibly for the pick-up to be positive when using the DWY approach while being negative using the SWY approach.

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[22] DWY has been defined slightly different than in formula 10. In formula 10 we used the elasticity of the bonds as weights here in formula 25 we are using the dollar duration (basis-point values) as weights.

[23] It has been the standard in Denmark (or maybe is the standard) to calculate pick-up on Barbells using the DWY approach, se for example Dahl (1994a) and Christensen (1995).

Using the SWY approach - as shown here - will lead to a pick-up calculation with is consistent with the naive yield-dynamics assumption (as also IRR is) - that is consistent with ROR. That is the SWY approach will therefore lead to pick-ups with is not misleading as is the case when using the DWY approach.

4: Conclusion

In this paper we have analysed three different kinds of Barbell strategies; Standard Barbell, Butterfly Barbells and Box Barbells.

This papers generel conclusion is the following:

Even though SWY is theoretical superior to DWY in the calculation of the expected return on a position under a naive yield-dynamics assumption - it does not behave well under in a dynamics yield-dynamics assumption.

The message here is: using SWY as the tool for measuring the pick-up on the combined position - should only be done if the position has been hedged against the implied -yield movements from formula 20. This mean for example that if we wish to speculate in the SWY on the Standard-Barbell we need to enter into a Box-Barbell strategy. But because we initially are trading the pick-up on the Box-Barbell strategy this kind of trade should only be entered if the SWY pick-up on the Standard-Barbell and the Box-Barbell have equal sign and optimal close to each other.

If we on the other hand wish to speculate in the DWY pick-up then things are more straight-forward - as proposition 3 shows that entering into a -Barbell means that you are trading the DWY pick-up of an -Barbell. The problem here if of course that the DWY pick-up gives misleading information about the expected return on the position.

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Appendix A

Table 1: Level-Factor

Left/Right Wing	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
-1.00	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47
-0.75	3.72	3.72	3.72	3.72	3.72	3.72	3.72	3.72	3.72
-0.50	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97
-0.25	4.22	4.22	4.22	4.22	4.22	4.22	4.22	4.22	4.22
0.00	4.47	4.47	4.47	4.47	4.47	4.47	4.47	4.47	4.47
0.25	4.72	4.72	4.72	4.72	4.72	4.72	4.72	4.72	4.72
0.50	4.97	4.97	4.97	4.97	4.97	4.97	4.97	4.97	4.97
0.75	5.22	5.22	5.22	5.22	5.22	5.22	5.22	5.22	5.22
1.00	5.47	5.47	5.47	5.47	5.47	5.47	5.47	5.47	5.47

Table 2: Slope-Factor

Left/Right Wing	-1.00	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00
-1.00	0.15	0.2	0.25	0.29	0.34	0.39	0.44	0.49	0.54
-0.75	0.1	0.15	0.2	0.25	0.29	0.34	0.39	0.44	0.49
-0.50	0.05	0.1	0.15	0.2	0.25	0.29	0.34	0.39	0.44
-0.25	0	0.05	0.1	0.15	0.2	0.25	0.29	0.34	0.39
0.00	-0.04	0	0.05	0.1	0.15	0.2	0.25	0.29	0.34
0.25	-0.09	-0.04	0	0.05	0.1	0.15	0.2	0.25	0.29
0.50	-0.14	-0.09	-0.04	0	0.05	0.1	0.15	0.2	0.25
0.75	-0.19	-0.14	-0.09	-0.04	0	0.05	0.1	0.15	0.2
1.00	-0.24	-0.19	-0.14	-0.09	-0.04	0	0.05	0.1	0.15

Table 3: Adjusted Relative Value of the Middle Bond (RVMB)

Left/Right Wing	-1.00	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00
-1.00	3.92	4.05	4.17	4.29	4.41	4.53	4.65	4.77	4.89
-0.75	4.05	4.17	4.3	4.42	4.54	4.66	4.78	4.9	5.02
-0.50	4.18	4.3	4.42	4.55	4.67	4.79	4.91	5.03	5.15
-0.25	4.31	4.43	4.55	4.67	4.8	4.92	5.04	5.16	5.28
0.00	4.44	4.56	4.68	4.8	4.92	5.05	5.17	5.29	5.41
0.25	4.57	4.69	4.81	4.93	5.05	5.17	5.3	5.42	5.54
0.50	4.7	4.82	4.94	5.06	5.18	5.3	5.42	5.55	5.67
0.75	4.83	4.95	5.07	5.19	5.31	5.43	5.55	5.67	5.8
1.00	4.95	5.08	5.2	5.32	5.44	5.56	5.68	5.8	5.92

Appendix B

Usually a second order polynomial approximation is used as a proxy for the price change in a given bond, ie:

$$P - P_o = k(r_O - r) + \frac{1}{2}c(r - r_o)^2 \tag{1}$$

Where P_o and r_o are respectively the existing price and associated yield on the bond. Alternatively P and r are respectively the new price after the yield has changed from r_o - r. Lastly k and c are the dollar duration and dollar convexity.

The dollar duration k and dollar convexity c are defined as:

$$k = \sum_{t=1}^{n} F_t t(1 + r_o)^{-t-1} \quad ; \quad \mathrm{c} = \sum_{\mathrm{t}=1}^{\mathrm{n}} \mathrm{F_t t(t+1)}(1 + r_o)^{-t-2} \tag{28}$$

In many situations the approximation given in formula 1 is quite accurate - even for relatively large yield changes. The approximation is however in general only valid for small yield-changes and only for larger yield-changes if the convexity is negligible.

Instead of showing at this point which kind of approximation errors we can encounter by using formula 1 we will without proof state the following new approximation formulas which are much more robust.[24]:

$$P = P_o e^{E\frac{\left(\frac{1+r}{1+r_o}\right)^{-G} - 1}{G}} \tag{29}$$

Where E is the elasticity, ie: $E = \dfrac{k(1 + r_o)}{P_o}$. G is defined as:

$$G = \frac{c(1 + r_o)^2}{P_o E} - E - 1 \tag{30}$$

In the case of a zero-coupon bond, formula 3 will degenerate into:

$$P = P_o \left(\frac{1 + r}{1 + r_o}\right)^{-E} \tag{31}$$

Formula 3 can be turned around, so we are able get a closed form solution for the yield on a bond for a given price change. This yields:

$$r = (1 + r_o)\left(1 - \frac{G}{E}\ln\left(\frac{P_o}{P}\right)\right)^{-\frac{1}{G}} - 1 \tag{32}$$

[24] This approximation formula was first derived by Dahl (1994b).

For a zero-coupon this can expressed as:

$$r = (1 + r_o)\left(\frac{P_o}{P}\right)^{-\frac{1}{E}} - 1 \tag{33}$$

In the following table the robustness of these new approximation formulas is shown:

Table 1:

Yield	Clean Price	Calculated Clean Price (Formula 3)	Price error	Calculated yield (Formula 6)	Yield error in basis-points
1,00	**239,19**	**242,78**	**3,59**	**1,08**	**8,00**
4,00	**148,47**	**148,67**	**0,20**	**4,01**	**1,00**
6,00	**113,06**	**113,07**	**0,01**	**6,00**	**0,00**
7,00	***99,94***	***99,94***	***0,00***	***7,00***	***0,00***
8,00	**89,06**	**89,05**	**-0,01**	**8,00**	**0,00**
10,00	**72,31**	**72,26**	**-0,05**	**9,99**	**-1,00**
13,00	**55,54**	**55,35**	**-0,19**	**12,96**	**-4,00**

P_0 is assumed to be represented by the grey row in table 1. In the example we have used the 7% INK St. Lån 24 and furthermore the calculation is done using 27 April 1998 as the trading date.

As is easily seen from table 1, the formula is indeed very robust - this kind of nearly negligable approximation errors is known not to be supported by the normal approximation formula represented by formula 1.

As a comparison we can mention that using formula 1 to forecast the price change for the yield-changes in table 1 gives rise to the following prices:

Table 2:

Yield	Clean Price	Calculated Clean Price (Formula 1)	Price error
1,00	**239,19**	**211,54**	**-27,65**
4,00	**148,47**	**145,72**	**-2,75**
6,00	**113,06**	**112,97**	**-0,09**
7,00	***99,94***	***99,94***	***0,00***

8,00	**89,06**	**89,13**	**0,07**
10,00	**72,31**	**74,20**	**1,89**
13,00	**55,54**	**68,50**	**12,96**

Appendix C

Table 1: Level-Factor

Left/Right Wing	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
-1.00	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47
-0.75	3.72	3.72	3.72	3.72	3.72	3.72	3.72	3.72	3.72
-0.50	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97	3.97
-0.25	4.22	4.22	4.22	4.22	4.22	4.22	4.22	4.22	4.22
0.00	4.47	4.47	4.47	4.47	4.47	4.47	4.47	4.47	4.47
0.25	4.72	4.72	4.72	4.72	4.72	4.72	4.72	4.72	4.72
0.50	4.97	4.97	4.97	4.97	4.97	4.97	4.97	4.97	4.97
0.75	5.22	5.22	5.22	5.22	5.22	5.22	5.22	5.22	5.22
1.00	5.47	5.47	5.47	5.47	5.47	5.47	5.47	5.47	5.47

Table 2: Slope-Factor

Left/Right Wing	-1.00	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00
-1.00	0.15	0.2	0.25	0.29	0.34	0.39	0.44	0.49	0.54
-0.75	0.1	0.15	0.2	0.25	0.29	0.34	0.39	0.44	0.49
-0.50	0.05	0.1	0.15	0.2	0.25	0.29	0.34	0.39	0.44
-0.25	0	0.05	0.1	0.15	0.2	0.25	0.29	0.34	0.39
0.00	-0.04	0	0.05	0.1	0.15	0.2	0.25	0.29	0.34
0.25	-0.09	-0.04	0	0.05	0.1	0.15	0.2	0.25	0.29
0.50	-0.14	-0.09	-0.04	0	0.05	0.1	0.15	0.2	0.25
0.75	-0.19	-0.14	-0.09	-0.04	0	0.05	0.1	0.15	0.2
1.00	-0.24	-0.19	-0.14	-0.09	-0.04	0	0.05	0.1	0.15

Table 3: Adjusted Relative Value of the Middle Bond (RVMB)

Left/Right Wing	-1.00	-0.75	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00
-1.00	3.65	3.69	3.73	3.77	3.82	3.86	3.9	3.94	3.99
-0.75	3.85	3.9	3.94	3.98	4.02	4.07	4.11	4.15	4.19
-0.50	4.06	4.1	4.15	4.19	4.23	4.27	4.32	4.36	4.4
-0.25	4.27	4.31	4.35	4.4	4.44	4.48	4.52	4.57	4.61
0.00	4.48	4.52	4.56	4.6	4.65	4.69	4.73	4.77	4.82
0.25	4.68	4.73	4.77	4.81	4.85	4.9	4.94	4.98	5.02
0.50	4.89	4.93	4.98	5.02	5.06	5.1	5.15	5.19	5.23
0.75	5.1	5.14	5.18	5.23	5.27	5.31	5.35	5.4	5.44
1.00	5.31	5.35	5.39	5.43	5.48	5.52	5.56	5.6	5.65

Appendix D

Table 1: Butterfly Barbell Strategy (Trading date: 27 April 1998)

	Left Wing	Middle Bond	Right Wing	
Bond	4% INK Stgb I	8 INK St.lån 06	7% INK St.lån	Combined Position
Clean Price	99.20	118.06	112.90	
Acc. Interest	0.83	1.00	3.21	
Price	100.03	119.06	116.11	
Yield	4.47	5.15	5.24	
Modified Duration	1.68	5.93	6.86	
Dollar Duration	1.68	7.06	7.97	7.06
Position	210.47	-100.00	44.33	
Value/Cash-payout	210.53	119.06	51.47	-142.94

Table 2: Box Barbell Strategy (Trading date: 27 April 1998)

	Left Wing	Middle Bond	Right Wing	
Bond	4% INK Stgb I	8 INK St.lån 06	7% INK St.lån	Combined Position
Clean Price	99.20	118.06	112.90	
Acc. Interest	0.83	1.00	3.21	
Price	100.03	119.06	116.11	
Yield	4.47	5.15	5.24	
Modified Duration	1.68	5.93	6.86	
Dollar Duration	1.68	7.06	7.97	7.06
Position	75.46	-100.00	72.76	
Value/Cash-payout	75.48	119.06	84.48	-40.90

Table 3: Return Simulation Matrix (Butterfly Barbell Strategy)

Left/Right Wing	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
-1.00	-0.07	0.53	1.12	1.72	2.3	2.89	3.47	4.05	4.62
-0.75	-0.63	-0.04	0.55	1.14	1.72	2.3	2.88	3.45	4.02
-0.50	-1.19	-0.6	-0.02	0.56	1.14	1.72	2.29	2.86	3.42
-0.25	-1.74	-1.16	-0.58	0	0.57	1.14	1.71	2.27	2.83
0.00	-2.29	-1.71	-1.14	-0.57	0	0.57	1.13	1.69	2.24
0.25	-2.83	-2.26	-1.69	-1.13	-0.56	0	0.55	1.11	1.66
0.50	-3.37	-2.81	-2.24	-1.68	-1.12	-0.57	-0.02	0.53	1.08
0.75	-3.9	-3.34	-2.79	-2.23	-1.68	-1.13	-0.58	-0.04	0.51
1.00	-4.43	-3.88	-3.33	-2.78	-2.23	-1.68	-1.14	-0.6	-0.06

Appendix E

Let now r(0,t) and r(0,T) for T > t represent the yields for the maturities t and T. It is now assumed that a portfolio consisting of w_1 units of the t-period zero-coupon bond and w_2 units of the T-period zero-coupon bond is constructed, for $1 = w_1 + w_2$. The question is now - what is the yield on this portfolio?[25]

The equation we have to solve is given by:

$$\frac{w_1}{(1 + r(0,t))^t} + \frac{w_2}{(1 + r(0,T))^T} = \frac{w_1}{(1 + r)^t} + \frac{w_2}{(1 + r)^T}$$

where r is the rate we are looking for.

This expression can be reformulated, because $(1 - r) \approx \frac{1}{(1 + r)}$ for r being "small", as:

$$w_1(1 - r(0,t))^t + w_2(1 - r(0,T))^T \approx w_1(1 - r)^t + w_2(1 - r)^T$$

Given that t and T are positive integers, it is known that:

$(1 - r)^n = 1 - \binom{n}{1} r^1 + \binom{n}{2} r^2 - \binom{n}{3} r^3 + ... \binom{n}{n} r^n$, where $\binom{n}{k} = \frac{n!}{(n - k)!k!}$ is the binomial coefficient.

This means that we now can express formula 2 as:

$$w_1(1 - \binom{t}{1} r(0,t).....etc) + w_2(1 - \binom{T}{1} r(0,T)....etc) \approx w_1(1 - \binom{t}{1} r....etc) + w_2(1 - \binom{T}{1} r...etc)$$

From this it can be deduced that the yield on the portfolio can approximately be formulated as:

$$r \approx \frac{w_1 t r(0,t) + w_2 T r(0,T)}{w_1 t + w_2 T}$$

Because the elasticity on a zero-coupon is equal to the maturity, we have now proved the second line in equation 10 in section 2.3 in the main text[26].

[25] It is of course always possible to use an iteration method in order to determine this yield.

[26] A similar proof is given in “Barbells -A Nordic Perspective” Appendix B.

Appendix F

Figure 1:

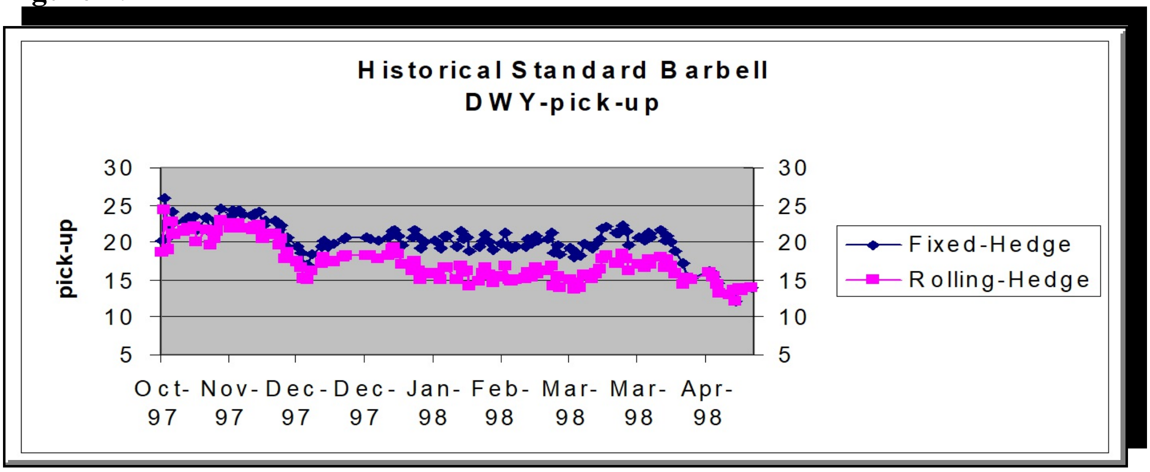

Figure 2:

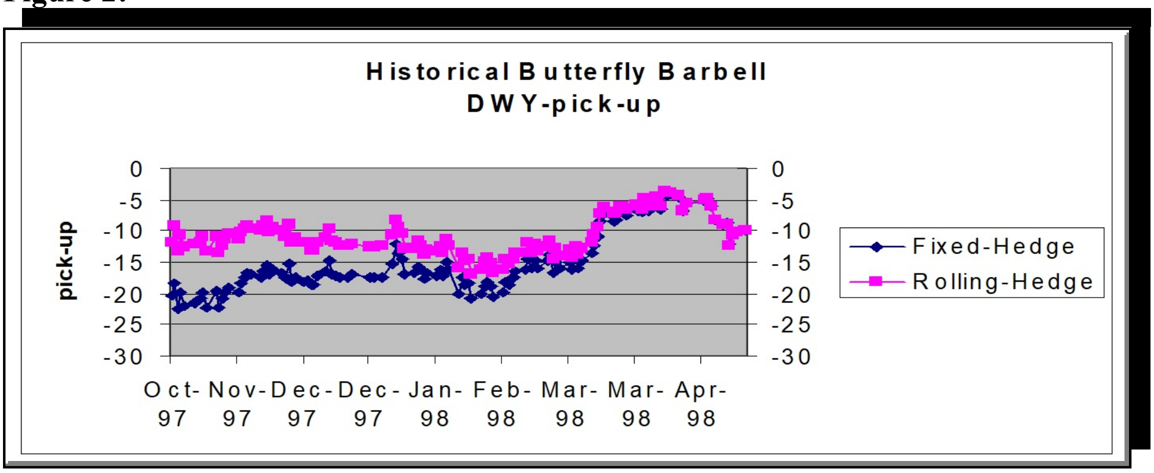

Figure 3: