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# Modelling Danish Mortgage Bonds

## Introduction

The purpose of this paper is to explain the background behind the key-figures for the danish mortgage bonds that are calculated on NASDAQ and at the same time give some insight into the methods employed.

First however we will start with a short general introduction to the mortgage bond market in Denmark, including its history.

The paper will be organized as follows: First we will make a short introduction to the products that exist in the Danish mortgage bond market. After that we will explain the key-figures that are being calculated and give an explanation of each of them. Lastly we will have the more technical section, where the methods/models will be explained.

## History

The issuance of callable mortgage bonds in Denmark dates back more than 200 years (1795). During the last 200 years the system has gone through a number of stages and survived several occasions of economic and political turmoil, including the bankruptcy of the Kingdom of Denmark in the 1813, the depression in the 1930s and the Financial Crisis of 2008 with no record of default.

The strong track record is mainly attributable to the strong legislative framework, which from a very early stage in the development of the market, has put great emphasis on the protection of the mortgage bond investors by imposing strict risk limits on the mortgage banks (the principle of balance). In 1850 the first Mortgage Bond Act was passed. The legal framework has been amended several times since, but the guiding principles such as the balance and investor protection principles have remained unchallenged.

Mortgage bonds are at present issued by a comparatively small number of mortgage banks - generally with identical characteristics - adding to the liquidity of the bonds issued. Furthermore, market concentration is high, with only two mortgage banks accounting for approximately two third of all mortgage bonds issued.

The Danish mortgage market is the largest in the world compared relative to GDP and the second largest in Europe in absolute terms.

In 2007, an amendment to the legal framework came into force offering universal banks access to covered bond funding alongside the established specialist mortgage banks. The amendment also had the purpose of rendering the Danish covered bond system compliant with the covered bond criteria in the EU Capital Requirement Directive.

The amendment introduced different bond types, three of which could be called covered bonds (as they fulfill UCITS and CRD). SDO (særligt dækkede obligationer), SDRO (særligt dækkede realkreditobligationer) and Realkreditobligationer issued before 31. of December 2007.

All of them are classified as covered bonds and are CRD compliant and thus carry low risk weights (10% according to the standardized approach). The only difference between the SDOs and SDROs is that SDROs may be issued by specialist mortgage banks only, whereas SDOs may be issued by both universal banks and specialist mortgage banks.

Finally, the amendment allowed the specialist mortgage banks to issue Realkreditobligationer but Realkreditobligationer issued after 31 December 2007 are not CRD compliant and high risk weights (20% according to the standardized approach) apply for these bonds relative to SDOs/SDROs. Furthermore, the amendment gave the specialist mortgage banks as well as the universal banks the possibility to issue under two different balance principles.

- The specific balance principle, which is very close to the old balance principle, where the loan to the household was matched exactly by the bond bought by the investor
- The general balance principle, which is more in line with what we see in other European countries

An important and central point in the classical mortgage system is the right for the householders to repay loans at par (or not significantly different from par). This repayment right was kept in the new SDO legislation, through an amendment to L 199. Both balance principles are used by the respective Danish Mortgage institutes (RD, DLR are issuing under the specific balance principle, while NYK and BRF are issuing under the general balance principle).

## Bond Types

The covered bond market in Denmark has experienced a profound transition over the past 15-20 years. Traditionally, fixed rate callable annuity bonds dominated the market, mirroring the callable fixed rate mortgage loans in the Danish property market. Non-callable bullet bonds were introduced to fund interest-reset loans (adjustable-rate loans), which were launched in 1996. Since then, a large demand for interest-reset loans has shifted the Danish covered bond market to such an extent that non-callable bullet bonds made up close to 50% of total market volume at the end of 2012. A return to the old traditional fixed rate callable bonds can however be seen in recent years, as seen in figure 1 below. The reason for this is fourfold:

- The very low interest rate environment, which led to the opening of fixed-rate callable bonds at 1.5% for the 30 year loan-segment in February 2015

- The new Danish covered bond legislation addressing refinancing risk, which was implemented 1. April 2014
- The Danish FSA "Supervisory Diamond for Mortgage Institutes" as from September 2014, which impose restrictions on interest rate risk, share of IO-loans and amount of loans with short funding
- The Basel/CRD IV/CRR measurement of "net stable funding requirement" (NSFR) as well as the implied guidance from the Rating bureau' used by Danish mortgage institutes have underpinned the ongoing transition from short to longer dated funding profiles

The above points have led to an increase in contribution margin for especially floating rate loans, this together with a low interest rate environment has been the factor driving this drift towards fixed rate callable mortgage loans.

Floating rate covered bonds (FRNs) with an embedded cap structure also met increasing demand. As a result, mortgage banks introduced a line of products in 2004 that were funded by issuing floating-to-fixed covered bonds or capped floaters. In 2005, FRNs without a cap (some with floors) were introduced, targeting corporate clients, and in 2007 FRNs with a ratchet coupon were launched (in Danish RenteDyk).

In February of 2015 the central bank lowered the deposit rate to an all time low of -0,75 pct. The reason was in order to keep a stable currency to EUR. This led to negative rates on short term non-callable bonds, the ones that are used for refinancing mainly 1Y interest reset loans. The mortgage banks was not ready for this and different solutions ware selected. In some cases a floor was imposed on the interest reset rate whereas others decided to pass the negative rates to the borrowers. None of these solutions are stable long term solutions, whereas a solution where the negative interest rate gives rise to extra ordinary repayments is much more natural, which also is a method under consideration.

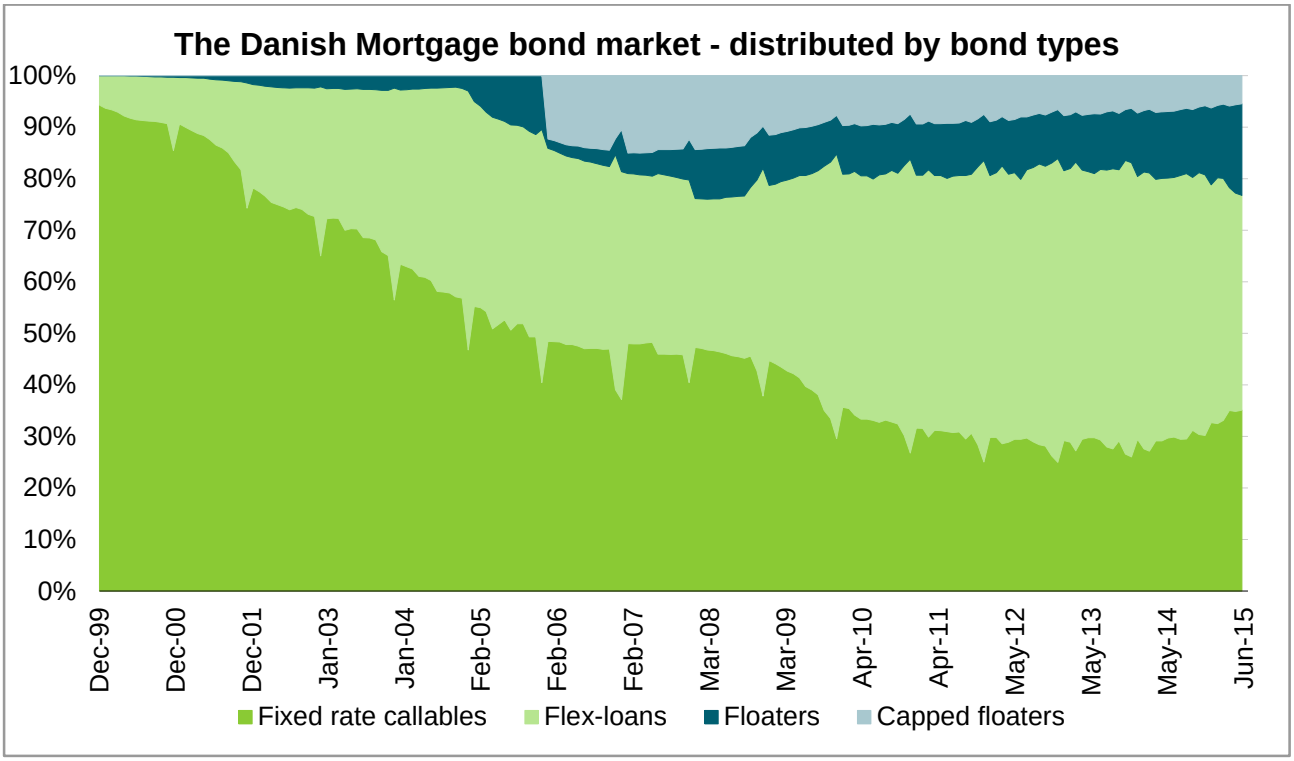


Figure 1.

## Callable annuity bonds

The fixed-rate, callable annuity bond is considered as being the traditional mortgage bond. This bond has a built-in Bermudan call option, which means that the borrowers may prepay their remaining outstanding debt at a price of 100 (par) at each repayment date during the life of the loan.

This Bermudan call option provides borrowers with a high degree of security. Without this option, the market price of – in this case a non-callable – bond could rise to much more than 100 if yields decrease meaning that borrowers could become technically insolvent.

For the investor (owner of the bond), this means a possible loss due to that a part of the nominal position can be prepaid at price 100 even though the price in the market may be higher. The percentage of prepayment is monitored closely each quarter and published on a weekly basis.

Borrowers' interest payments and redemptions made on the payment dates are being distributed to investors in accordance with the percentage of bonds repaid so that any investor's holding in a given bond series will be written down corresponding to the overall percentage of bonds repaid in that series. The amount is rounded to the nearest øre (DKK 0.01) for bonds denominated in Danish

kroner and euro cents for bonds denominated in euro. The percentages of bonds repaid and the corresponding amount are published on the publication date.

Due to harmonisation of the EU bond market (corporate actions (CA)) and the implementation of a joint European clearing system (TARGET2) a change in the repayment rules for Danish mortgage bonds has been implemented. For most Danish mortgage bonds this has effect from the payment date 1. October 2015, more specifically for all bonds that has a payment after the 21th of September 2015. Which means that Denmark is part of TARGET2 when it is launched the 12th of September 2016.

The main difference being that between the publication-date (which remains unchanged – approximately 6 weeks before the payment data for bonds with 4-payments per year) and the payment-date, the trading of mortgage bonds is done inclusive the repaid amount. The repaid amount is first being separated from the bond at the payment-date. The final repaid amount will be fixed at the publication-date, whereas the repayment percentage can change from the publication-date to the payment-date. This will however only occur if the outstanding amount of the bond changes between these two dates, which in general only will occur for bonds open for issuing.

## Non-callable bullet bonds

Non-callable bullet bonds are fixed rate bonds with a single annual payment. Maturities range from 1 to 11 years, with emphasis on the 1- to 5-year segment. The characteristics of the bonds mirror those of plain-vanilla Danish government bonds and most European covered bonds.

As is the case for callable bonds, the loans that are interest-reset are repaid in accordance with the ordinary annuity or annuity with an interest-only option.

As the bonds funding the loans are bullet bonds, the bonds and loans needs to be rebalanced on a regular basis which in most cases is once a year (it however depends on the type of loan, length of refinancing period) by issuing an amount of bonds required to offset the remaining principal of the annuity profile of the individual loan.

## Floating-to-Fixed, Capped Floaters and Ratchets

Floating-to-Fixed and Capped floaters are a line of floating rate products with embedded caps (and possible floors) applied to the entire maturity of the loans (max 30 years). These loans are offered both as annuity and as annuity with a 10-year interest-only option exercisable during the term of the loan at the borrower's discretion.

Interest rates for the DKK-denominated bonds are fixed semi-annually/quarterly based on the six/three month CIBOR plus a fixed margin. However, interest payments and redemptions fall due quarterly.

In a number of cases the bond in question has an embedded repayment option that allows the borrower to repay the loan at 100/105 (normally 105).

In principle two different cap structures are available. Floating-to-Fixed is based on a floating-to-fixed cap structure, whereby interest rates will become fixed at the cap rate if the cap is triggered (a knock-in-option). In contrast capped floaters are based on a traditional cap structure where interest rates are floating for the entire maturity of the loan, albeit maximized at the cap rate.

In contrast to this Ratchets has a floating cap-rate which means that the next coupon rate can never be higher than the previous one – and furthermore in case the next coupon rate is lower than the previous the cap-rate is being set to that rate.

Most of the bonds in this category are of the type Capped Floaters. All floating-to-fixed bonds was knocked-in to become traditionally fixed rate callable bonds no later than 24th of September 2008 and the last Ratchet was opened for issuing at 10 of April 2008.

### Floating-rate bonds

The floating rate covered bond market is very diversified and the bonds have a range of different characteristics. The majority of the floating rate bonds are denominated in DKK or EUR with interest rate fixing against 3M EURIBOR and 3M/6M CIBOR/6M Cita, respectively. They can be callable or non-callable, have annuity or bullet profiles and have 2 or 4 terms per year.

## Keyfigures

The following key-figures are being calculated:

| Keyfigure | Description |
|---|---|
| **Zero Prepayment price (ZPP)** | The present value of the cash flow assuming ordinary repayments..<br><br>More precisely, ZPP is calculated using the estimated yield-curve. ZPP is the non-callable dirty price.<br><br>Definition 1: |

| | |
|---|---|
| | The dirty price (DP) is defined as:<br>$$DP = CP + ACC$$<br>Where CP is the clean price and ACC is the accrued interest rate at the settlement-date.<br><br>For estimating the yield-curve either Danish or EUR swaps are being used. |
| **OA Price (OAP)** | The option adjusted price of the bond calculated using the employed stochastic term structure model and prepayment model, for more see below.<br><br>OAP is a dirty price. |
| **OA Spread (OAS)** | The spread which makes OAP match the market dirty price – measured in BP (basis points) |
| **OA PVBP (OAPVBP)** | The absolute change in OAP if the yield-curve is chocked by one basis point.<br><br>Is being calculated as a centered difference, more precisely as follows:<br>$$OAPVBP = \frac{OAPVBP\_UP - OAPVBP\_DOWN}{2}$$<br>As both OAPVBP_UP and OAPVBP_DOWN are absolute risk-measures, we have that OAPVBP is an absolute risk-measure, meaning that it represents an absolute price-change |
| **OA Duration (OAD)** | OAD is defined as a percentage of the initial dirty price (OAP), so OAD is a relative risk-measure.<br><br>It is being calculated as a centered difference, more precisely as follows:<br>$$OAD = -\frac{OAP(Up) - OAP(Down)}{2xOAPx0.01}$$ |

| | |
|---|---|
| | OAD is what is termed a Modified Duration measure.<br><br>OAP(Up) and OAP(Down) are explained below. |
| **OA Convexity (OAC)** | Option adjusted convexity. The change in OAP - used in calculating OAC – is +/-10 basis points. OAC is as OAD defined as a relative risk-measure.<br><br>It is being calculated as:<br><br>$$OAC = \frac{[OAP(+10BP) + OAP(-10BP)] - 2xOAP}{0.01xOAPx\left(\frac{10}{10000}\right)^2}$$ |
| **OA PVBP Up (OAPVBP_UP)** | The absolute change in OAP if the yield-curve is chocked up-ward by one basis point.<br><br>Is calculated as:<br><br>$$OAPVBP\_UP = -OAP(Up) - OAP$$<br><br>Where OAP(Up) is the option adjusted dirty price if the yield-curve is chocked up-ward by one basis point. |
| **OA PVBP Down (OAPVBP_DOWN)** | The absolute change in OAP if the yield-curve is chocked down-ward by one basis point.<br>Is calculated as:<br><br>$$OAPVBP\_DOWN = -OAP(Down) - OAP$$<br><br>Where OAP(Down) is the option adjusted dirty price if the yield-curve is chocked down-ward by one basis point. |
| **Option Free Yield (Yield)** | YTM calculated assuming ordinary repayments (see below) |
| **Weighted Average Life (WAL)** | Weighted Average Life. Is calculated from the Mortgage Model, and are defined as followed: |

| | |
|---|---|
| | $$WAL = \frac{\sum_{t=1}^{T} tH(t)}{\sum_{t=1}^{T} H(t)}$$<br><br>Where H(t) is the re-payment at time t. Actually we have that H(t) is the expected re-payment at time t – as we do not have just a single re-payment at time t but in the prepayment model have a whole range of possible re-payments at time t. |
| **Model prepayment rate (MPR)** | Prepayment rate forecast (for the next 'unpublished' term date (from prepayment model).<br><br>For more details see below. |

## Which bonds are being calculated for?

Calculations require that the mortgage bond is a fixed rate callable bond.

Given that is fulfilled, the following rules apply:

- There has to be an Opening-Date
- There has to be a Closing-Date
- Cash flow is available, either synthetic or delivered. By synthetic is meant the cash flow generated by NASDAQ when a bond is open for issuing. By delivered is meant the cash flow that origins from the Mortgage Banks and "only" contain known loans
- Bond has a positive outstanding amount
- Issuer List is the following:
  20,32,47,49,50,51,57,58,63,64,65,67,68,69,71,72,73,74,75,77,78,79,81,84,85,86,87,89,90,91,92,93,95,97

## Fixed-Rate Bonds – Technical section

Any model for the pricing of fixed rate callable bonds contains two main ingredients:

- A stochastic interest rate model to model the evolution in the yield-curve
- A prepayment-model for projecting future repayments

## The stochastic Interest Rate Model

The stochastic interest model used is the so-called Hull-White model, more precisely the following version:

(1.1) $$dr(t) = [\theta(t) - \kappa r(t)]dt + \sigma(t)dW$$

Where σ(t) – the instantaneous volatility - is piecewise constant. The mean-reversion κ is assumed to be a constant. The parameter θ(t) is time-dependent and is uniquely determined by the given yield-curve, which is determined from swap-prices – either denominated in DKK or EUR. The volatility structure is specific through the parameters κ and σ(t), which are being derived from Swaption prices - either denominated in DKK or EUR.

The numerical implementation is a trinomial tree with three branches going from each node. The lattice is constructed in such a way, that nodes coincide with payment-dates – in order to not introduce numerical approximation errors.

## Factors influencing the prepayment behavior

Compared to American callable mortgage bonds there are the following two main differences:

- In America the call provision only allow prepayment of mortgages at par value, it is not as in Denmark possible to buy the bond in the market at the market value and deliver these to the mortgage institution to cancel out
- In America new homeowners cannot take over an existing mortgage (this is possible in Denmark, the difference is due to the fact that in Denmark the loan follows the house, whereas in America it follows the person). This means that prepayments in America will be related to the house-turnover

In Denmark the prepayment incentive is mostly interest-rate dependent. There exist however three cases which can have a big impact on prepayment – and which are not solely interest-rate related:

- Change of tax-rules
- Media campaign
- Issuing of new (re)financing products

The mortgage credit institutions provide information to investors via NASDAQ.

The table below is a list of the information available:

| Type of Data | TIP-Transaction | Description of TIP |
|---|---|---|

| | | |
|---|---|---|
| CK91: Cash flows used for yield calculation (delivered or calculated) | BasicDataBondCashFlow | Publish both delivered and synthetic cash flows and cash flows delivered for informative purposes only |
| CK94: Cash flow information open series (delivered cash flows on open series which weren't used for yield calculation) | BasicDataBondCashFlow | Publish both delivered and synthetic cash flows and cash flows delivered for informative purposes only |
| CK92: Composition of debtors | BasicDataDebtorComposition | Publish information on debtor composition |
| CK93: Extraordinary redemptions | BasicDataExtraRedemption | Publish information on pre-payments |
| CK95: Notification of drawings and Notification of drawingpercentage | BasicDataNotificationDrawing and BasicDataNotificationDrawingPercent | Publish information about drawn amounts and drawing percentage respectively |

## The debtor distribution

Due to the fact that the group ”Private” (private homeowners) over time has become a more professional player in this game (the game of house refinancing ) there is no reason to assume that there is any significant difference between the behavior for the average ”Private” borrower and the average ”Other” borrower.

However, a difference in prepayment speed is inherent in the historical prepayments when comparing loan-sizes. For that reason we from the debtor distribution data construct a model with “only” 2-debtor groups:

- One consisting of all loans with a remaining principal below 3 million DKK
- One that contains all loans with a remaining principal above 3 million DKK

For that reason we have that in the prepayment model the total prepayment rate for a given bond is given by:

(1.2)
$$\Gamma = \sum_{i=1}^{2} w(i)\lambda(i)$$

Where w(i) and λ(i) is the weight and prepayment rate for group i, for i ∈ [1,2]. Furthermore we have that the weights sum to one (1). The weights are being calculated using the (constructed outstanding bond debt for each of the 2 groups.

## Determining the refinancing-rate

As prepayment behavior is mainly interest rate dependent it is very important to have a sound and robust procedure for finding the refinancing rate.

The specification of the refinancing-rate is linked to the assumed issuing pattern for new mortgage bonds. As a rule of thumb it is assumed that the debtor refinancing structure is a stepwise linear function in the following 4 segments:

- Segment 1: $0 < T \leq 10$
- Segment 2: $10 < T \leq 15$
- Segment 3: $15 < T \leq 20$
- Segment 4: $20 < T \leq 30$

Where T is the remaining time to maturity of the existing loan. This means that if for example the remaining time to maturity of the existing loan is 17 years the refinancing rate will be determined using the refinancing rate of Segment 3.

The refinancing rates is assumed to be flat in each of the segments.

From a modeling point of view the prepayment model do not work directly with for example a 30-year fixed rate callable mortgage bond. Instead, proxy instruments are being created along the following lines:

- For each of the segments a proxy annuity bond is created with the same characteristic as the mortgage bonds that are open for issuing in the given Segment
- A refinancing spread is estimated that ensures that the average price for mortgage bonds that are open for issuing in the given Segment, and the price calculated using the yield-curve for the proxy annuity bond are identical. This refinancing spread is assumed constant for each Segment

## Published prepayments – how are they included in the Prepayment Model?

Published prepayments puts a lower bound on the estimated Prepayment Rate for the payment dates where information is available.

To limit discontinuity in OAD - and especially OAC – the following procedure is employed:

- Prepayment forecast for the first payment date is performed purely in the model up until (for mortgage bonds with quarterly payments) 2 month before the payment-date – the loan repayment notification date
- Between the loan repayment notification date and the publications date, the estimated prepayment rate is estimated by weighting the debtor distribution repayment-rate with the prepayment rate implied from the model
- After the publication date, forecast for the first payment date is identical to the repayment-rate of the prepayments

It is however worth pointing out here, namely that the keyfigure MPR – when we pass the publication date will now not contain the prepayment rate for the first coming payment date but instead for the next one.

## Delivered or Synthetic cash flow?

All calculations are performed using the synthetic cash flow.

## The Prepayment Model

The prepayment function is dependent on:

- The debtor distribution – as explained above
- The current and lagged refinancing-rates for the relevant Segments
- The poolfactors (see definition below) for each of the 2 debtor groups
- Other variables are:
    - The remaining maturity on the existing loan
    - The coupon on the existing loan – a weighted coupon
    - The short rate. This in order to allow the model to incorporate refinancing to 1Y interest-rest loans

As a rule of thumb the prepayment function consist of 2 distributions. The truncated normal distribution is used for calculating the basis repayment rate, which are then scaled by the poolfactor distribution. For “low” poolfactor bonds the prepayment model converges to a type of CPR-model (CPR = Conditional/Constant Prepayment Rate).

Definition 2:

Poolfactor is defined as follows:

(1.3) $$poolfactor = \frac{CO}{OO}$$

Where CO is the outstanding principal balance and OO is the original outstanding principal balance.

Definition 3:

CPR measures prepayments as a percentage of the current outstanding loan balance.

The

## Pricing principle

The pricing method used is a semi Monte Carlo model, which consist of the following main steps:

- Construct the lattice for each of the 4 Segments – refinancing rates
- Use the prepayment model at each node, rolling forward in the lattice, to derive the expected prepayment at each node
- Discount the derived cash flow at each node to find the price

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