

# CVA – a look behind the scenes

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12. June 2012



# What is counterparty risk?

- The risk taken on by an entity entering an OTC contract with a counterparty having a relevant default probability. As such, the counterparty might not respect its payment obligations.
  - The counterparty credit risk is defined as the risk that the counterparty to a transaction could default before the final Settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default

# CVA – what is it?

- CVA stands for Credit Valuation Adjustment and is a price adjustment
- CVA is obtained by pricing the counterparty risk component of a deal, similarly to how one would price a credit derivative
- Its counterpart DVA: Debit Value Adjustment – more later....



# What does CVA depend on?

- CVA is a function of:
  - The OTC contract's underlying volatility
  - The correlation between the underlying and default of the counterparty
  - The counterparty credit spreads volatility
- What is WWR?
  - Is the amplified risk when the reference underlying and the counterparty are strongly correlated in the wrong direction

# CVA...a bit more

- Collateral
  - Are we risk-free?
- Netting
- Basel III
  - Basel noticed that during the financial crisis only one third of losses due to counterparty risk were due to actual defaults. The remaining losses have been due to CVA mark-to-market losses
    - => FOCUS AREA

# Unilateral versus Bilateral

- CVA is a positive quantity, an adjustment to be subtracted from the default-risk free price in order to account for the counterparty default risk in the valuation
- Unilateral ( $\Rightarrow$  agree Bank cannot default):
  - $UCVA(\text{Bank}) = CDVA(\text{Corporate})$
  - $UDVA(\text{Bank}) = UCVA(\text{Corporate}) = 0$
- Bilateral formula for the value of the OTC derivative:

$$\text{Price(Risk-Free)} + DVA - CVA$$

and

$$BVA = DVA - CVA$$

# General Formula – unilateral case - I

- The general formula for the value of a contract as seen from the point of the bank can be expressed as:

$$W_B^D(t, T) = 1_{\tau_c > T} W_B(t, T)$$

$$+ 1_{t < \tau_c \leq T} [W_B(t, \tau_c) + P(t, \tau_c) [R_c [PV_B(\tau_c)]^+ - [-PV_B(\tau_c)]^+]]$$

- The last expression seen from the Company is:

$$[PV_c(\tau_c)]^+ - R[-PV_c(\tau_c)]^+$$

# Explanation of general formula

- If there is not early default, this formula simplifies to value of a default-free claim
- If there is early default of the counterparty, the payments that occur before default are received
- And then if the residual present value is positive only the recovery ( $R = 1 - \text{LGD}$ ) is received
- Whereas if it is negative it is paid in full by the Bank

## General Formula – unilateral case - II

- Taking the risk neutral expectation, one can obtain the fundamental formula for the valuation of counterparty risk when the Bank is default free:

$$E_t[W_B^D(t, T)] = 1_{\tau_c > T} [W_B(t, T)] - E_t[LGD_c 1_{t < \tau_c \leq T} P(t, \tau_c) [PV_B(\tau_c)]^+]$$

- Where:  $PV(\tau_c) = E_t[W(\tau_c, T)]$  is the value of the transaction on the counterparty default date
- WWF – where is that?

# The case of symmetry...

- There are 6 possible events:

$$A = \tau_B \leq \tau_c \leq T$$

$$B = \tau_B \leq T \leq \tau_c$$

$$C = \tau_c \leq \tau_B \leq T$$

$$D = \tau_c \leq T \leq \tau_B$$

$$E = T \leq \tau_B \leq \tau_c$$

$$F = T \leq \tau_c \leq \tau_B$$

## General Formula – bilateral case

- The general formula for the value of a contract in the case of symmetric counterparty risk is:

$$\begin{aligned}
 W_B^D(t, T) &= 1_{E \cup F} W_B(t, T) \\
 &+ 1_{C \cup D} [W_B(t, \tau_c) + P(t, \tau_c) [R_c [PV_B(\tau_c)]^+ - [-PV_B(\tau_c)]^+]] \\
 &+ 1_{A \cup B} [W_B(t, \tau_B) + P(t, \tau_B) [[PV_B(\tau_B)]^+ - R_B [-PV_B(\tau_B)]^+]]
 \end{aligned}$$

- Can be expressed as:

$$E_t[W_B^D(t, T)] = E_t[W_B(t, T)] - CVA_B(t) + DVA_B(t)$$

$$CVA_B(t) = E_t[LGD_c 1_{t < \tau(\text{first}) = \tau_c < T} P(t, \tau_c) [PV_B(\tau_c)]^+]$$

$$DVA_B(t) = E_t[LGD_B 1_{t < \tau(\text{first}) = \tau_B < T} P(t, \tau_B) [-PV_B(\tau_c)]^+]$$

$$\text{for } 1_{A \cup B} = 1_{t < \tau(\text{first}) = \tau_B < T} \text{ and } 1_{C \cup D} = 1_{t < \tau(\text{first}) = \tau_c < T}$$

# Strange consequences of the formula new term DVA

- Credit quality of investor WORSENS  $\Rightarrow$  books POSITIVE Mark-To-Market
- Credit quality of investor IMPROVES  $\Rightarrow$  books NEGATIVE Mark-To-Market
- Example:
  - Citigroup in its press release on the first quarter revenues of 2009 reported a positive mark to market due to its worsened credit quality: “Revenues also included [...] a net 2.5 \$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads”

# Bilateral...First-To-Default

- Bilateral adjustment as seen from Bank is:  
 $BVA(B) = DVA(B) - CVA(B)$  – and opposite seen from the Company due to symmetry!
- Why first-to-default?
  - In computing DVA and CVA one need to account for both defaults of Bank and Corporate in both terms. This means that effectively there is a first-to-default check. This can be explained as:
    - If the bank is doing the calculations, in scenarios where the bank defaults first the DVA term will be activated and the CVA term vanishes, whereas in scenarios where the corporate defaults first then the bank DVA vanishes and the bank CVA payoff activates....some banks ignores this!

# DVA or no DVA?

- NO DVA: Basel III, page 37, July 2011 release
  - This CVA loss is calculated without taking into account any off setting debit valuation adjustments which have been deducted from capital Under paragraph 75
- YES DVA: FAS 157
  - Because non performance risk (the risk that the obligation will not be fulfilled) includes the reporting entitys credit risk, the reporting entity should consider the effect of its credit risk (creditstanding) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements FAS157 (see also IAS39)



## First-to-default CVA and First-to-default DVA

- The first-to-default CVA is consistent from an accounting standpoint. However, this definition does not come without shortcomings:
  - One problem is that, if the credit of the computing party is low, the FTDCVA can be substantially lower than the UCVA. At default, the FTDCVA even vanishes. These material discrepancies between the FTDCVA and the UCVA contemplated in the Basel III Accord have undesirable side effects, such as for instance giving a competitive advantage to those financial institutions which are the slowest at endorsing the banking reform



## FTDCVA and FTDDVA - continued

- A second problem is that the FTDCVA is unhedgeable since, to hedge it, a bank would have to short its own credit, an impossible trade. This aggravates the already serious problem of unhedgeability of the UDVA term as it appears in the definition of the BCVA
- Finally, if the FTDCVA is used for pricing while the UCVA is used to determine capital requirements, the material mismatch between the two makes it difficult to optimize risk management strategies



# Basel III

- The Basel III Accord prescribes that banks should compute UCVA by assuming independence of exposure and default. The advanced framework allows banks to implement the effect of WWF in the calculation of their exposures by using own models, while under the standardized approach the Basel III Accord accounts for the effect by means of a one-size-fits-all multiplier
- Basel III accord chooses to ignore the UDVA in the calculation for capital adequacy requirements. Although consideration of the UDVA needs to be included for accounting consistency, no such principle exists as far as capital requirements are concerned



# CVA – practical issues..

- Due to the variety of possible different definitions of CVA (Unilateral CVA, Bilateral CVA, first-to-default CVA) combined with boundary conditions such as risk free or replacement close outs and the option to use either exact or approximate treatment of WWF, there appears to be material discrepancies in CVA valuation across financial institutions.
- The most significant discrepancy is between UCVA and FTDCVA. Both are used in the industry for valuation, while only the former is endorsed for regulatory purposes in Basel III.

# Closeout: Replication (ISDA?) versus Risk Free

- We have not mentioned it explicitly but in the formulas given earlier we assumed a risk free PV upon the first default, to close the deal (our PV's)
- ISDA (2009) Close-out Amount Protocol:
  - "In determining a Close-out Amount, the Determining Party may consider any relevant information, including, [...] quotations (either firm or indicative) for replacement transactions supplied by one or more third parties that may take into account the credit worthiness of the determining party at the time the quotation is provided"
- Brigo and Morino (2010) "Dangers of Bilateral Counterparty Risk: the fundamental impact of close out conventions", available at [ssrn.com](http://ssrn.com)

# Payoff Risk

- The exact payout corresponding with the Credit and Debit valuation Adjustment is not clear
  - DVA or not?
  - Which Close out?
  - First-to-default risk or not?
  - How are collateral and funding accounted for exactly?
    - Funding Risk!
- This is Worse than model risk:
  - Payout risk. WHICH PAYOUT?



# A general Pricing Framework

- Assumption: The Bank enters a transaction with a counterparty and, when dealing with Unilateral Risk, the Bank Considers it self default free
- We model and calibrate the default time of the counterparty using a stochastic intensity default model
- We model the transaction underlying and estimate the deal PV at default.
- We allow for the counterparty default time and the contract Underlying to be correlated

# The case of independency

- In case we assume independency between the discount factor (the yield-curve model) and the default probabilities we get:

$$E_t[W_B^D(t, T)] = Q(\tau_c > T)[W_B(t, T)] - LGD_c Q(t < \tau_c \leq T) E_t[P(t, \tau_c)[PV_B(\tau_c)]^+]$$

- Next slide...the case of stochastic intensity

# Stochastic Intensity

- One nice candidate is the CIR++ model, which looks like:

$$\lambda_t = y_t + \psi(t; \kappa, \mu, \nu, y_0)$$

for

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dZ_t$$

$$\tau_c = \Lambda^{-1}(\varepsilon) \text{ and } \Lambda(T) = \int_0^T \lambda(s)ds$$

- Where:  $\psi(t; \dots)$  is used to match the initial CDS-Curve and  $\varepsilon$  is the standard exponential independent of all brownian motions

# Example: IRS

- The price can be expressed as:

$$IRS^D(t, T) = IRS(t, T) - LGD_c Q(t < \tau_c \leq T) Swaption(\tau_c, \dots)$$

- A potential model is the following:
  - A 1-factor HW model to model the dynamic in interest rates
  - A 1-factor CIR++ model to model default
  - These 2 stochastic processes are assumed to be correlated

# Stop for now!

- More examples....at another time 😊